UNCLASSIFIED

AD NUMBER
ADA800180
CLASSIFICATION CHANGES
TO: **unclassified**
FROM: **confidential**
LIMITATION CHANGES
TO: **Approved for public release; distribution is unlimited.**
FROM: **Distribution authorized to DoD only; Administrative/Operational Use; JUL 1949. Other requests shall be referred to Aeronautical Systems Div., Wright-Patterson AFB, OH 45433. Pre-dates formal DoD distribution statements. Treat as DoD only.**
AUTHORITY
ASD/ENF ltr dtd 9 Mar 1992; ASD/ENF ltr dtd 9 Mar 1992

THIS PAGE IS UNCLASSIFIED

Theoretical Investigation of the Dynamic Stability of Bomber-Fighter Coupled Flight

Prepared in Connection with

Phase I of Air Force Project MX-1016

- CONFIDENTIAL -

This document contains information affecting the national defense of the United States within the meaning of the Espionage Act, (U.S.C. 50:31,32). The transmission of this document or the revelation of its contents in any manner to any unauthorized person is prohibited.

Prepared by: H. R. Lu

S. Helfman

J. Cangelosi

Approved by:

F. J. Mulholland
Asst. Chief Aerodynamics

W. A. O'Donnell
Chief Development Engr.

A. Kartveli
Chief Engineer

TABLE OF CONTENTS

SUMMARY

The problem of dynamic stability of attaching two fighters to the wing tips of a bomber was considered in detail. In particular, the case of two F-84 airplanes on a B-50 bomber was investigated.

The equations of motions were derived for two methods of attachment, single-joint, and two-joint attachments. Due to the complexity of the problem mathematically, distrubed motions were treated independently in pitch, in roll and in yaw. The stability of the assumed system was checked by the Routh-Hurwitz Discriminants.

The elevator and the ailerons were considered in the single-joint attachment to constitute the auto-control in the fighters. In the two-joint attachment, ailerons or flaps were moved symmetrically to provide change of wing lifts on fighters. The movement of the control deflection in response to signals of both the amplitude and the rate of change of flapping angle about the hinge or joint was assumed to be instantaneous. Effect of varying auto-pilot parameters, and relative c.g. locations from the hinge axis was studied.

It was found that stability of the single-joint attachment was marginal if auto-controls in the fighters only were allowed. Inclusion of auto-pilot control in the bomber or even manuel control from bomber pilot greatly improved stability of the coupled flight system. Stability of the two-joint attachment was found to be positive even when auto-control in fighters only were considered. The relative merit of the two methods of attachment can not be weigned until loads at the joints are determined.

It is pointed out that the analysis made in this report was done on the basis of rigid airplanes. The effect of elastic deformation or flexibility of the wing structure, which presents another mode of vibration, may be critical from the standpoint of both the stability of the combination and the strength of the bomber wing. This phase of the problem should be considered in the near future.

Dynamic Stability of Bomber-Fighter Coupled Flight

Introduction

This report deals with the problems of dynamic stability of attaching two fighters to the wing tips of a bomber for the purpose of increasing the range of the escorts. Dr. Vogt in Ref. 1 showed that the benefit derived from the increased aspect ratio of the bomber-fighter team practically counter-balanced the additional drag of the fighters. Consequently, the range of the bomber was not thought to be impaired. It was assumed that this result was generally accepted, and it remained to show theoretically that the combined flight was feasible.

In the free-flight tunnel of the NACA, a simplified test, fundamentaly similar to the bomber-fighter combination, was made. The fighters in the form of tip wings were attached to the bomber in such a way that pitching and yawing of the fighters relative to the bomber were restrained. It was demonstrated that stable flight was attained only when flaps on the wing tip wings were linked so as to move in ~~the~~ proportion to the angular displacement of the tip wings relative to the main wing. In the analysis contained herein it will be shown that stability is easily achieved with such an attachment.

In this report, stability analysis was made for two methods of attachment. These are: (a) single-joint attachment, and (b) two-joint attachment. In the single-joint attachment, the fighters are free to pitch and to roll, and partially free to yaw. Each fighter is attached to the wing tip of the bomber at one point only by a socket and lance arrangement. This method of attachment has the advantage of small torsional load introduced in the bomber wing if the joint is located near the elastic center of the wing structure. Control of fighters to fly in alignment with bomber wing may be accomplished by standard control surfaces of the fighters. In the two-joint attachment, the fighters are free to roll or flap about the wing tip joint only. Since the fighters are restrained in pitch and yaw, larger loads would be expected on the bomber wing. Controls may be afforded by moving flaps, flaps and spoilers, or by symmetric movement of the ailerons. Clearly, some redesign or re-rigging of the control surface systems is necessary in such cases.

It will be shown that with either methods of attachment, the system of bomber-fighter team is unstable without auto-pilot control in the fighters. For stability, elevator and aileron deflections proportional to the wing misalignment angle and the rate of change of wing misalignment angle were considered. The effects of center of gravity locations of the fighters and of the bomber relative to the axis of rotation of the fighters were also treated.

In writing the equations of motions, it was assumed that the speed and the altitude of the bomber remained essentially constant. It was also assumed that the pitch, the roll, and the yaw motions can be treated independently. Only the linear part of the differential equations were retained for stability analysis. In a separate report, the motions of the airplanes in gust disturbance were computed taking into account the non-linear terms. These non-linear terms come principally from the centrifugal forces. It will be shown that these non-linear terms are negligible but improve stability slightly rendering the analysis presented herein somewhat on the conservative side.

Notations and Symbols

The standard definitions and symbols used in aerodynamics have been retained in this report. The XYZ axes with origin at the center of gravity of the bomber are a set of orthogonal axes fixed to the body of the bomber; the xyz axes with origin at the center of gravity of the fighter, fixed to the body of the fighters.

m = mass of the fighter airplane, slugs

I = Polar moments of inertia, slug - ft.^{2}

Subscripts with capital letters refer to the bomber
lower-case letters to the fighter

k = radius of gyration, ft.

R = Bomber wing - semi-span, ft.

r = Fighter wing - semi-span, ft.

a_1 = Distance from flapping axis of the fighter to the lift vector of the fighter wing, ft.

A = Distance from bomber pitching axis (c.g.) to the fighter pitching axis (the joint hinge), ft.

c = Distance from fighter pitch axis (hinge) to c.g. of fighter, ft.

d = Distance from MACenter of the fighter wing to fighter c.g., ft.

l_t = tail length of the fighter, ft.

ϕ = roll angle of the bomber wing-plane from horizontal plane of the space axes, rad.

β = flap angle of the fighter wing-plane from the bomber wing-plane, rad. (relative coordinate)

θ = pitch angle of the bomber wing-plane from the horizontal plane of the space axes, rad.

α	=	pitch angle of the fighter wing-plane from the horizontal plane of the space axes, rad.
ψ	=	yaw angle of the bomber wing from the vertical plane XZ of the space axes, rad.
γ	=	relative yaw angle of the fighter from the bomber, rad.
$\sum L_w$	=	Summation of incremental fighter wing lift caused by disturbed motion, lbs.
$\sum L_t$	=	Summation of incremental fighter tail lifts caused by disturbed motion, lbs.
L_o	=	External rolling disturbance on combination, ft. lbs.
L_1	=	External rolling disturbance on fighter about hinge, ft. lbs.
M_{γ_1}	=	External pitching disturbance due to anti-symmetric gust on fighter, ft. lbs.
M_o^1	=	External pitching disturbance on combination, ft. lbs.
M_o	=	External pitching disturbance on bomber, ft. lbs.
M_2	=	External pitching disturbance due to symmetric gust on fighter, ft. lbs.
L_2	=	External rolling disturbance on fighter, ft. lbs. occurring with pitch motion.
N_o	=	External yawing disturbance on combination, ft. lbs.
N_1	=	External yawing disturbance on fighter, ft. lbs.
M_{F_α}	=	Fighter fuselage pitching moment due to angle of attack change, ft. lbs./rad.
S	=	Fighter wing-area, ft.^{2}
V	=	Level flight speed, ft./sec.
q	=	Dynamic pressure, lbs./sq. ft.
δ	=	Control deflection, rad. (NACA convention of signs)
k	=	Auto-pilot rate control ratio
K	=	Auto-pilot displacement control ratio

Subscripts:

w - wing t - tail

e - elevator f - flap

a - ailerons

Subscripts θ, $\dot{\theta}$, β, $\dot{\beta}$, α, $\dot{\alpha}$ etc denote partial differntiation with respect to the variables, as $L_{\dot{\phi}}$, $L_{\dot{\beta}}$ etc., etc. $= \frac{\partial L}{\partial \dot{\phi}}$, $\frac{\partial L}{\partial \dot{\beta}}$ etc., etc.

Positive direction for angles, velocities and accelerations, and distances of the forces are indicated in Fig. 2 which shows schematically the two proposed methods of coupling considered.

Flight Condition for Investigation

Appendix A contains data on the cruising altitude and true airspeed for the best range of B-50 airplane, which is to be used to form the bomber-fighter team with two F-84 airplanes. Fig. 1 shows the cruise speed as a function of the altitude for cruise. It is unfortunate that no information was made available on the actual range of the bomber as a function of the cruise speed and cruise altitude. It is conceivable that the stability of the coupled flight would be more critical when the cruise altitude was high because the aerodynamic damping and restoring force would be relatively smaller than the inertia forces. For the purpose of analysis, an altitude of 25,000 ft. and a cruise speed of 300 mph (260 knots) true were chosen.

Mass Data:

	B-50	F-84
Design Gross Weight, lbs.	120,000	14,600
Design Mass, Slugs	3,730	$m =$ 453.4
Moment of Inertia, slug-ft.^{2}		
I_{xx}	$1,712 \times 10^3$	14,280.3
I_{yy}	$1,000 \times 10^3$	18,594.8
I_{zz}	$2,712 \times 10^3$	31,717
Radius of Gyrations, ft.		
k_x		5.61
k_y		6.41
k_z		8.36

		B-50	F-84
Physical Dimensions - Pertinent Data:			
Semi-span, ft.		R = 71.7	r = 18.3
Wing Area, $ft.^2$		1720	S = 260
MAC		12.87	c = 7.39
Tail Length, ft.		51	l_t = 18.14
Design L.G. Location:	% MAC	26	28
From Hinge at Joint, ft.		A = 3.25	e = .5912
Neutral Point in Pitch :	% MAC	34	34
Wing Center of Pressure Location	% MAC	25	25
Distance from c.g.: ft.			d = .2217

$$V = 440 \text{ ft/sec}$$

$$q = 103.5 \text{ lb/ft}^2$$

Aerodynamic Forces and Moments

In estimating aerodynamic forces and moments on the bomber and the fighters, an investigation of the effect of mutual interference among the airplanes due to induced flow phenomena of the combined planform of the wing was made and the results are reported in Ref. 2. This investigation is concerned only with the lift forces on the airplanes due to various symmetric and anti-symmetric modes of flight. The aerodynamic moments of the fuselages were assumed to be not influenced by the induced flow.

1. Fighter Wing Lift - L_w

(a) Due to Change of Angle of Attack of Fighter Wing, α

$$L_{w_\alpha} \cdot \alpha = \frac{\partial L_w}{\partial \alpha} = qS \frac{\partial C_{L_w}}{\partial \alpha} \alpha$$

The $\partial C_{L_w} / \partial \alpha$ is obtained from the spanwise lift calculation by considering the B-50 bomber at fixed attitude and varying the angle of attack of the fighter F-84 as a symmetric twist configuration.

$$\frac{\partial C_{L_w}}{\partial \alpha} = 4.439 \text{ per radian (Ref. 2, p. 22)}$$

$$L_{w_\alpha} \cdot \alpha = 4.439 \times 103.5 \times 260 \, . \, \alpha$$

$$= 119.4 \times 10^3 \, . \, \alpha \text{ - lbs.}$$

The location of the center of pressure from the wing tip joint is:

$$a_{w_\alpha} = 17.95 \text{ ft.} \qquad \text{(Ref. 2, p. 22)}$$

(b) <u>Due to Change of Angle of Attack of Bomber Wing</u>, θ

$$L_{w_\theta} \cdot \theta = q \ S \ \frac{\partial C_{L_w}}{\partial \theta} \cdot \theta \cdot$$

From similar calculations in Ref. 2, p. 21

$$L_{w_\theta} \cdot \theta = 103.5 \times 260 \times .8455 \ . \ \theta$$

$$= 22.8 \times 10^3 \ . \ \theta \ - \text{lbs.}$$

The corresponding lever arm from the joint is

$$a_{w_\theta} = 9.67 \text{ ft.}$$

(c) <u>Due to Rolling Velocity of the Bomber</u>, $\dot{\phi}$

$$L_{w_{\dot{\phi}}} = -92.91 \ \frac{q}{V} \qquad \text{(Ref. 2, p. 12)}$$

$$= -92.91 \times \frac{103.5}{440} = -21.85 \times 10^3 \text{ lb./rad.}$$

The corresponding arm is

$$a_{w_{\dot{\phi}}} = 17.62 \text{ ft.} \qquad \text{(Ref. 2, p. 12)}$$

(d) Due to Rolling Velocity of the Fighter, $\dot{\beta}$

$$L_{w_{\dot{\beta}}} = -20.71 \frac{q}{V} \qquad \text{(Ref. 2, p. 12)}$$

$$= -20.71 \times \frac{103.5}{440}$$

$$= -4.871 \times 10^3 \text{ lb./rad./sec/}$$

The corresponding arm is

$$a_{w_{\dot{\beta}}} = 21.6 \text{ ft.}$$

The fighter wing lift, therefore, is:

For roll motion

$$L_w = (119.4\,\alpha - 21.85\,\dot{\phi} - 4.871\,\dot{\beta}) \times 10^3 \text{ lb.} \qquad \text{Eq. I}$$

For pitch motion

$$L_w = (119.4\,\alpha - 4.871\,\dot{\beta} + 22.8\,\theta) \times 10^3 \text{ lbs.} \qquad \text{Eq. II}$$

2. Fighter Tail Lift - L_t

(a) Due to Angle of Attack of Fighter, α

$$L_{t_\alpha} = CN_{t_\alpha} \left(1 - \frac{d\epsilon}{d\alpha}\right) \eta_t \cdot q \cdot S_t$$

$$= 3.49\,(1 - .47) \times .95 \times 103.5 \times 48.3$$

$$= 8.784 \times 10^3 \text{ lbs./rad.}$$

(b) <u>Due to Downwash Lag</u>, $\dot{\alpha}$

$$L_{t_{\dot{\alpha}}} = C_{N_{t_\alpha}} \cdot S_t \cdot l_t \cdot \eta_t \cdot \frac{d\epsilon}{d\alpha} \cdot q/V$$

$$= 3.49 \times 48.3 \times 18.14 \times .95 \times .47 \times 103.5/440$$

$$= 321.1 \text{ lb./rad./sec.}$$

(c) <u>Due to Damping-in Pitch of Bomber</u>, $\dot{\underline{\theta}}$

$$L_{t_{\dot{\underline{\theta}}}} = C_{N_{t_\alpha}} \cdot q \cdot S_t \cdot \frac{l_t}{V} \cdot \frac{K}{\sqrt{\eta_t}}$$

$$= 3.49 \times 103.5 \times 48.3 \times \frac{18.14}{440} \times \frac{1.1}{\sqrt{.95}}$$

$$= 811.4 \text{ lb/rad./sec.}$$

In the roll motion, it may be shown that

$$\dot{\underline{\theta}} = \dot{\alpha} + \frac{\ddot{Z}}{V} = \dot{\alpha} + \frac{R + r}{V}\ddot{\phi} + \frac{r}{V}\ddot{\beta}$$

$$= \dot{\alpha} + .2045\,\ddot{\phi} + .04159\,\ddot{\beta}$$

(d) <u>Due to Rolling of Bomber</u>, $\dot{\phi}$

$$\Delta\alpha_t = -[R + r] \cdot \frac{\dot{\phi}}{V}$$

$$L_{t_{\dot{\phi}}} = -C_{N_{t_\alpha}} \cdot q \cdot S_t \cdot \frac{R + r}{V}$$

t

$$= -3.49 \times 103.5 \times 48.5 \times \frac{90}{440}$$

$$= -3.568 \times 10^3 \text{ lb./rad./sec.}$$

(e) Due to Flapping of Fighter, $\dot{\beta}$

$$L_{t_{\dot{\beta}}} = -C_{N_{t_{\alpha}}} \cdot S_t \cdot q \cdot \frac{\ell}{V_t}$$

$$= -3.49 \times 48.3 \times 103.5 \times 18.3/440$$

$$= -725.5 \text{ lb/rad./sec.}$$

(f) Due to Elevator Control Deflection, δ_e

$$L_{t_{\delta_e}} = C_{N_{t_{\delta_e}}} \cdot S_t \cdot q$$

$$= 1.89 \times 48.3 \times 103.5 \qquad (\alpha_{\delta_e} = .538)$$

$$= 9.448 \times 10^3 \text{ lb./rad.}$$

The fighter tail lift, therefore, is:

For the roll motion:

$$L_t = L_{t_\alpha}\alpha + L_{t_{\dot{\alpha}}}\dot{\alpha} + L_{t_{\dot{\theta}}}\dot{\theta} + L_{t_{\dot{\phi}}}\dot{\phi} + L_{t_{\dot{\beta}}}\dot{\beta} + L_{t_{\delta_e}} \cdot \delta_e$$

$$= 8.784 \times 10^3 \alpha + 1.1325 \times 10^3 \dot{\alpha} + 165.9 \ddot{\phi} + 33.75 \ddot{\beta}$$
$$-3.568 \times 10^3 \dot{\phi} - 725.5 \dot{\beta} + 9.448 \times 10^3 \delta_e \qquad \text{Eq. III}$$

For the pitch motion:

$$L_t = 8.784 \times 10^3 \alpha + 1.1325 \times 10^3 \dot{\alpha} + .03375 \times 10^3 \ddot{\beta}$$
$$- .7255 \times 10^3 \dot{\beta} + 9.448 \times 10^3 \delta_e \qquad \text{Eq. IV}$$

3. Damping-in Roll Moments

(a) Due to Rolling of the Combination, $\dot{\phi}$

$$L_{\dot{\phi}} = -31.56 \times 10^6 \; q/V \qquad \text{(Ref. 2, p. 10)}$$

$$= -31.56 \times 10^6 \times 103.5/440$$

$$= -7.423 \times 10^6 \text{ ft. lbs./rad./sec.}$$

(b) Due to Flapping of the Fighter, $\dot{\beta}$

$$L_{\dot{\beta}} = -4.021 \times 10^6 \; q/V \qquad \text{(Ref. 2, p. 11)}$$

$$= -4.021 \times 10^6 \times 103.5/440$$

$$= -.94574 \times 10^6 \text{ ft. lbs./rad./sec.}$$

4. Fighter Fuselage Moment in Pitch

The fuselage moment of the fighter may be determined by the neutral point of the fighter at 34% MAC which is assumed for the condition of flight. For c.g. location of 28% MAC

$$M_{F_\alpha} = q\,S\,c\left[\left(\frac{d\,C_M}{d\,C_L} - \frac{d}{c}\right) C_{L_\alpha} + C_{N_{t_\alpha}} \cdot \frac{S_t}{S} \cdot \frac{l_t}{c}\left(1 - \frac{d\,\epsilon}{d\,\alpha}\right)\eta_t\right]$$

$$= 198.9 \times 10^3 \left[(-.06 - .03)\, 4.438 + 3.49 \times \frac{48.3}{260} \times \frac{18.14}{7.39} \times (1 - .47)\, .95\right]$$

$$= 79.86 \times 10^3 \text{ ft. lb./rad./sec.}$$

5. Static Restoring and Damping Moments of Bomber in Pitch

(a) Restoring Moment, M_θ

The restoring moment of the bomber is determined from the neutral point location of the B-50 airplane at 34% MAC (Appendix A). For a design c.g. location of the B-50 at 26% MAC,

$$M_\theta = q\,S\,c \cdot C_{m_\theta} = q\,S\,c\,\left[.26 - .34\right]\,C_{L_\theta}$$

$$= 103.5 \times 1720 \times 12.89 \times -.08 \times 5.101$$

$$= -.93478 \times 10^6 \text{ ft. lb./rad.}$$

(b) Damping -in- Pitch, $M_{\dot\theta}$

Appendix A contains data on B-50 airplane from Boeing Aircraft Company on damping-in-pitch.

$$M_{\dot\theta} = q \,.\, S \,.\, c \cdot C_{m_{\dot\theta}}$$

$$C_{m_{\dot\theta}} = \frac{-93.5}{v \text{ mph}} = \frac{137}{V \text{ ft./sec.}}$$

$$\therefore M_{\dot\theta} = 103.5 \times 1720 \times 1289 \left(-\frac{137}{440}\right)$$

$$= -.71337 \times 10^6 \text{ ft. lbs./rad./sec.}$$

6. Fighter Aileron Rolling Moment

$$L_{\delta_A} = C_{l_{\delta_a}} \cdot q \cdot S \cdot b$$

$$= .1146 \times 103.5 \times 260 \times 36.5$$

$$= 112.56 \times 10^3 \text{ ft. lb./rad.}$$

7. Yawing Moments of the Bomber

(a) Static Stability Moment, $N_{\psi_{B-50}}$

In the absence of wind tunnel data on the bomber, the yawing moments of the bomber were estimated:

Estimated $C_{N_{\psi_{B-50}}} = -.1775$ per rad. ($-.0031$ per deg.)

$$N_{\psi_{B-50}} = q \quad S \quad b \quad C_{N_\psi}$$

$$= 103.5 \times 1720 \times 141.5 \times (-.1775)$$

$$= -4.465 \times 10^6 \text{ ft. lb./rad.}$$

(b) Damping Moment, $N_{\dot{\psi}_{B-50}}$

$$\Delta\psi = \frac{l_t}{V}\dot{\psi}$$

Since the directional stability of an airplane comes principally from the vertical tail surface, it may be assumed that

$$N_{\dot{\psi}_{B-50}} = \frac{l_t}{V} \cdot N_{\psi_{B-50}}$$

$$= \frac{51}{440} (-4.465 \times 10^6)$$

$$= -0.518 \times 10^6 \text{ ft. lbs/rad./sec.}$$

8. <u>Yawing Moment of the Fighter</u>

From the wind tunnel data of the F-84 airplane (Ref. 3), it is found that

$$C_{n_\psi} = -.1318/\text{rad. } (-.0023 \text{ per degree})$$

$$C_{n_r} = .170/\text{rad./sec. } (-.00297 \text{ per deg. per sec.})$$

$$N_{\psi_{F-84}} = C_{n_\psi} \; q \; S \; b$$

$$= -.1318 \times 103.5 \times 260 \times 36.5$$

$$= -.1298 \times 10^6 \text{ ft. lbs./rad.}$$

$$N_{\dot{\psi}_{F-84}} = C_{n_r} \cdot q \; S \cdot \frac{b^2}{2V}$$

$$= -.17 \times 260 \times \frac{36.5^2}{2} \cdot \frac{103.5}{440}$$

$$= -.00696 \times 10^6 \text{ ft. lbs./rad.}$$

$$\Sigma N_{\dot{\psi}} = -.518 \times 10^6 + 2 \times (-.00696) \times 10^6 = -.532 \times 10^6$$

$$\Sigma N_{\psi} = -4.465 \times 10^6 + 2 \times (-.1298) \times 10^6 = -4.725 \times 10^6$$

CASE A - Single-Joint Attachment

Derivation of Equations of Motion

1. Roll Stability

Degrees of Freedom: (dependent variables)

Roll angle of bomber: ϕ

Roll angle of fighter: β

Pitch angle of fighter: α

Since all variables are angles, the equations of motion are moment equations. In writing these equations, one equates inertia moment to moment due to external forces. The external forces are due to disturbances and also aerodynamic forces. The inertia moments are due to mass of the fighter, m, moving relative to bomber body axis which in turn move relative to space axes. One may write down the components of the inertias as the vector sum of relative, entrained and complimentary accelerations by the theorem of Corislis or one may write Lagranges equation using ϕ, β, α as generalized coordinates.

The location of the center of mass of the fighter, P in Fig. 2, is determined for given ϕ and β by:

$$x = R \cos \phi + r \operatorname{Cos} (\phi + \beta) \tag{1}$$

$$y = R \sin \phi + r \sin (\phi + \beta) \tag{2}$$

Their time derivatives are:

$$\dot{x} = -R\dot{\phi}\sin\phi - r(\dot{\phi}+\dot{\beta})\sin(\phi+\beta)$$

$$\dot{y} = R\dot{\phi}\cos\phi + r(\dot{\phi}+\dot{\beta})\cos(\phi+\beta)$$

Therefore,

$$v^2 = \dot{x}^2 + \dot{y}^2 = R^2\dot{\phi}^2 + r^2(\dot{\phi}+\dot{\beta})^2 + 2Rr\dot{\phi}(\dot{\phi}+\dot{\beta})\cos\beta \text{ - - - - - } (3)$$

The Kinetic Energy, T, of the system is

$$T = \frac{1}{2} I_X\dot{\phi}^2 + mR^2\dot{\phi}^2 + m(k_x^2+r^2)(\dot{\phi}+\dot{\beta})^2 + 2mRr\dot{\phi}(\dot{\phi}+\dot{\beta})\cos\beta \text{ - - } (4)$$

In ϕ Equation,

$$\frac{\partial T}{\partial \dot{\phi}} = I_X\dot{\phi} + 2mR^2\dot{\phi} + 2m(k_x^2+r^2)(\dot{\phi}+\dot{\beta}) + 2mRr(2\dot{\phi}+\dot{\beta})\cos\beta$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\phi}}\right) = I_X\ddot{\phi} + 2mR^2\ddot{\phi} + 2m(k_x^2+r^2)(\ddot{\phi}+\ddot{\beta}) + 2mRr(2\ddot{\phi}+\ddot{\beta})\cos\beta$$
$$- 2mRr(2\dot{\phi}\dot{\beta}+\dot{\beta}^2)\sin\beta$$

$$\frac{\partial T}{\partial \phi} = 0$$

Therefore,

$$\left\{I_X + 2m(k_x^2+R^2+r^2+2Rr\cos\beta)\right\}\ddot{\phi} + 2m\left\{k_x^2 + r(R\cos\beta + r)\right\}\ddot{\beta}$$
$$-2mRr(2\dot{\beta}\dot{\phi}+\dot{\beta}^2)\sin\beta = \left[\begin{array}{c}\text{External \& Aerodynamic}\\ \text{Moments}\end{array}\right] \text{ - - - - } (5)$$

In β - equation ,

$$\frac{\partial T}{\partial \dot{\beta}} = 2m(k_x^2 + r^2)(\dot{\phi} + \dot{\beta}) + 2mRr\dot{\phi}\cos\beta$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\beta}}\right) = 2m(k_x^2 + r^2)(\ddot{\phi} + \ddot{\beta}) + 2mRr\ddot{\phi}\cos\beta - 2mRr\dot{\phi}\dot{\beta}\sin\beta$$

$$= 2m\left[k_x^2 + r(R\cos\beta + r)\right]\ddot{\phi} + 2m(k_x^2 + r^2)\ddot{\beta}$$

$$- 2mRr\dot{\phi}\dot{\beta}\sin\beta$$

$$\frac{\partial T}{\partial \beta} = -2mRr(\dot{\phi}^2 + \dot{\phi}\dot{\beta})\sin\beta$$

Therefore

$$2m\left[k_x^2 + r(R\cos\beta + r)\right]\ddot{\phi} + 2m(k_x^2 + r^2)\ddot{\beta} + 2mRr\dot{\phi}^2\sin\beta$$

$$= \left[\text{External and Aerodynamic Moments}\right]$$

- - - - - - - - - - - (6)

Bomber-Roll Equation (ϕ Equation)

There are several ways with which the ϕ - equation may be derived. It may be obtained by considering the system in its entirety or it may be obtained by considering the reaction force at the joint from the fighters. Because of the manner with which the aerodynamic damping moment are computed by spanwise lift calculation of the combined planform, it is convenient to consider the system in its entirety. It may easily be shown that by subtracting the β - equation from the ϕ - equation, the bomber roll equation may be reduced to that obtained by considering the hinge reaction at the joint.

For small deflections of ϕ and β,

$$\{I_0 + 2m[k_x^2 + (R+r)^2]\}\ddot{\phi} + 2m[k_x^2 + r(R+r)]\ddot{\beta} - 2mRr(2\dot{\phi}\dot{\beta} + \dot{\beta}^2)\cdot\beta$$
$$= L_0 + L_{\dot{\phi}}\dot{\phi} + L_{\dot{\beta}}\dot{\beta} + 2(R+r)\Sigma(L_w + L_t) \text{ ------- } (7)$$

where I_o = roll disturbance moment

$L_{\dot{\phi}}$ = damping-in-roll

L_w = Fighter wing-lift due to disturbance

L_t = Fighter tail-lift due to disturbance

Fighter Flapping Equation - β-equation

$$m[k_x^2 + r(R+r)]\ddot{\phi} + m(k_x^2 + r^2)\ddot{\beta} + mRr\dot{\phi}^2\cdot\beta$$
$$= L_1 + \Sigma L_{w_i} a_{w_i} + r\Sigma L_t \text{ ---------- } (8)$$

where a_i = distance the wing-lift acts from hinge point.

Fighter Pitch Equation - equation

Referring to Fig. 3, it is seen that

$$I_y \cdot \ddot{\alpha} = L_w d + R_F \cdot c + M_F - L_t \cdot l_t \qquad \text{- - - - - - - (9)}$$

Tp express this equation in dependent variables only, it is necessary to write the expression for the reaction force at the hinge.

$$R_F = -\sum (L_w + L_t) + mr\ddot{\beta} + m(R\cos\beta + r)\ddot{\phi} + mR\dot{\phi}^2 \sin\beta$$

For small angles,

$$R_F = -\sum (L_w + L_t) + mr\ddot{\beta} + m(R + r)\ddot{\phi} + mR\dot{\phi}^2 \beta \qquad \text{- - - - - - (10)}$$

Substituting,

$$I_y\ddot{\alpha} - mc(R+r)\ddot{\phi} - mcr\ddot{\beta} - mcR\dot{\phi}^2\beta$$
$$= M_1 + (d + c)\sum L_w - (c + l_t)\sum L_t + M_{F_\alpha} \cdot \alpha$$
$$\text{- - - - - - (11)}$$

It will be noted that the airplanes are assumed to be initially balanced and trimmed, i.e., the wing lift on the fighter equals to its weight, etc. Hence, the gravity term does not appear in the equations.

2. Pitch Stability

Degrees of Freedom:

Pitch of Bomber : θ

Roll of Fighters : β

Pitch of Fighters: α

Here, the reaction method is used in deriving the equations of motion. It is noted that in this case θ is the absolute angle, and $\alpha-\theta$ is the relative angle. In Fig. 3, all the acceleration or mass forces are shown by the theorem of Coriolis. It is assumed that the airplanes are separated at this joint. At this joint, there is a reaction force which may be found by summing up all the forces, inertia, external and aerodynamic, in the direction of the Y axis of the space axes. Dropping external force for the time being, one writes

$$R_F = m\ddot{\beta}r - m(c\ddot{\alpha} + A\ddot{\theta}) - mr\dot{\beta}^2\beta + m(c\dot{\alpha}^2 + A\dot{\theta}^2)\alpha - \Sigma(L_w + L_t) \qquad \text{- - - - - - - - - - (12)}$$

where A = c.g. location of bomber forward of the hinge

c = c.g. location of fighter aft of the hinge.

Bomber Pitch Equation - θ - equation

$$I_{YY}\ddot{\theta} = M_o + 2R_F \cdot A + M_\theta \cdot \theta + M_{\dot{\theta}} \cdot \dot{\theta}$$

Substituting R_F in above equation, one obtains

$$(I_{YY} + 2mA^2)\ddot{\theta} - 2mAr\ddot{\beta} + 2mAC\ddot{\alpha} + 2mA[r\dot{\beta}^2\beta - (c\dot{\alpha}^2 + A\dot{\theta}^2)\alpha]$$
$$= M_o + M_\theta \cdot \theta + M_{\dot{\theta}}\dot{\theta} - 2A\Sigma(L_w + L_t) \qquad \text{- - - - - - - - - (13)}$$

Fighter Flapping Equation β - equation

Taking moment about the wing-tip joint of all the forces on the fighter, one finds

$$m(k_x^2+r^2)\ddot{\beta} = \sum L_w a_w + r\sum L_t + m(A+C)r\ddot{\theta} + m(\ddot{\alpha}-\ddot{\theta})cr$$
$$-m\left[(A+C)\dot{\theta}^2 + C(\dot{\alpha}-\dot{\theta})^2 + 2C(\dot{\alpha}-\dot{\theta})\dot{\theta}\right]r\alpha + L_2$$

where L_2 = external disturbance moment on fighter.

Therefore

$$m(k_x^2+r^2)\ddot{\beta} - mAr\ddot{\theta} - mcr\ddot{\alpha} + m\left[A\dot{\theta}^2 + c\dot{\alpha}^2\right]r\alpha$$
$$= L_2 + \sum L_w a_w + r\sum L_t \quad \text{(14)}$$

Fighter Pitch Equation α - equation

Taking moment about the center of gravity of the fighter, remembering the reaction force excludes load due to external forces which effect are included in M_2.

$$I_y\ddot{\alpha} = d\sum L_w - l_t\sum L_t + R_F c + M_F + M_2$$

Substituting Eq. (12) in above, one obtains

$$(I_y+mc^2)\ddot{\alpha} + mAC\ddot{\theta} - mcr\ddot{\beta} + mc\left[r\dot{\beta}^2\beta - (c\dot{\alpha}^2 + A\dot{\theta}^2)\alpha\right]$$
$$= M_2 + M_{F\alpha}\alpha + (d-c)\sum L_w - (c+l_t)\sum L_t$$

$$\text{(15)}$$

3. <u>Yaw Stability</u>

Degrees of Freedom:

Yaw of Bomber : ψ

Yaw of Fighter : γ

This case is over-simplified. The reason for this simplification is that a system of four degrees of freedom would have to be considered since the roll or flapping of the fighter is always accompanied by pitch of the fighter. In view of the time limit of the contract, and of the lack of electronic computing device it is felt that this simplification is necessary.

It is also believed that although the yaw stability considered does not represent the actual flight, it is indicative of the stability of of the system. It will be seen that this system assumed is inherently stable. Therefore, rudder control is not believed necessary.

The equations of motion are similar to the case of roll stability. They are:

<u>Combined Yaw-Equation</u>:

$$\left\{I_{zz} + 2m\left[k_z^2 + (R+r)^2\right]\right\}\ddot{\psi} + 2m\left[k_z^2 + r(R+r)\right]\ddot{\gamma} - 2mRr(2\dot{\psi}\dot{\gamma} + \dot{\gamma}^2)\gamma$$
$$= N_0 + \Sigma N_{\psi}\psi + \Sigma N_{\dot{\psi}}\dot{\psi} + 2N_{\psi_{F\text{-}84}}\gamma + 2N_{\dot{\psi}_{F\text{-}84}}\dot{\gamma} \quad ----- (16)$$

<u>Fighter Yaw Equation</u>:

$$m\left[k_z^2 + r(R+r)\right]\ddot{\psi} + m(k_z^2 + r^2)\ddot{\gamma} + mRr\dot{\psi}^2\gamma = N_1 + N_{\psi_{F\text{-}84}}(\psi+\gamma) + N_{\dot{\psi}_{F\text{-}84}}(\dot{\psi}+\dot{\gamma}) \quad --- (17)$$

In these equations, ψ is the yaw angle of bomber, and γ is the relative yaw angle of the fighter away from the bomber. See Fig. 4.

CONFIDENTIAL

Stability Criteria

The consideration of stability is made by treating the homogeneous part of the simultaneous differential equations; i.e. when there is no forcing function or zero external disturbance. In general, the so called characteristic equation is of sixth degree for a system of three-degrees of freedom and is of the form

$$a_0D^6 + a_1D^5 + a_2D^4 + a_3D^3 + a_4D^2 + a_5D + a_6 \quad 0 \text{ - - - - - -} (18)$$

For stability, it is necessary for the above polynominal to have no root of positive real part. The criteria for the above condition is generally given by testing the so called Routh Hurwitz discriminants (Ref. 4).

$$\begin{pmatrix} a_1 & a_3 & a_5 & 0 \\ a_0 & a_2 & a_4 & a_6 \\ 0 & a_1 & a_3 & a_5 \\ 0 & a_0 & a_2 & a_4 \end{pmatrix}$$

Therefore, the criteria are:

$$a_1 > 0$$

$$a_1 a_2 - a_0 a_3 > 0$$

$$a_1 a_4 - a_0 a_5 > 0$$

$$a_3 (a_1 a_2 - a_0 a_3) - a_1 (a\, a_4 - a_0 a_5) > 0$$

$$a_5 (a_1 a_2 - a_0 a_3) - a_1^2 a_6 > 0$$

$$a_2 a_3 (a_1 a_4 - a_0 a_5) - a_0 a_1 a_3 a_6 > 0$$

These coefficients are considered as constant, the numerical values of which depend on the aerodynamic damping and restoring forces and moments as well as on the auto-pilot control parameters. For all the cases considered in this report, the criteria above are treated as a function of the design parameter of the auto-pilots and of the center of gravity locations.

Stability Analysis of the Coupled Flight of B-50 and F-84 Airplanes

In the preliminary investigation, it was found that stability of the coupled flight without automatic control was not positive. The analysis presented here include, (a) the basic stability without dynamic coupling; i.e., the center of gravity of the airplane are in line with the axis of rotation, (b) the effect of the locations of the center of gravity of the bomber and of the fighters, and (c) the effect of the automatic-pilot in the bomber airplane. The auto-pilot controls in the fighter airplanes are assumed to be instantaneously responsive to the deviation of the mis-alignment angle at the joint according to the following laws:

Elevator Deflection

$$\delta_e = k_e \dot{\beta} + K_e \beta \qquad (20)$$

Aileron Deflection

$$\delta_a = k_a \dot{\beta} + K_a \beta \qquad (21)$$

The elevator deflection generates pitch rotation for producing lift force on the fighter through change of angle of attack so as to displace the center of gravity or mass of the fighter. Whereas, the aileron deflection may supply the rotational moment so as to reduce the vertical shear load transmitted to the bomber wing at the joint.

1. Roll Stability

From Equations (7), (10) and (11) it is seen that the effect of the c.g. location of the fighter, "c" may be investigated separately since only the fighter-pitch equation (11) involves "c". Similarly, the auto-control in the bomber modifies only the bomber-roll equation (7).

(a) Basic Stability - No Dynamic Coupling

Substituting the expressions for aerodynamic forces and moments in Eq. (7), the bomber-roll equation becomes

$$\left[I_c + 2m\left(k_x^2 + \overline{R+r}^2\right)\right]\ddot{\phi} + 2m\left[k_x^2 + r(R+r)\right]\ddot{\beta} - 2mRr\left(2\dot{\phi}\dot{\beta} + \dot{\beta}^2\right)\beta$$
$$= L_o + L_{\dot{\phi}}\dot{\phi} + L_{\dot{\beta}}\dot{\beta} + 2(R+r)\sum\left(L_w + L_t\right) + 2L_{\delta_a}\delta_a$$

$$\left[1.712\times10^6 + 2\times453.4\left(\overline{5.61}^2 + \overline{90}^2\right)\right]\ddot{\phi} + 2\left[\overline{5.61}^2 + 90\times18.3\right]\ddot{\beta}$$
$$-2\times453.4\times71.7\times18.3\left(2\dot{\phi}\dot{\beta} + \dot{\beta}^2\right)\beta = L_o - 7.423\times10^6\dot{\phi} - .94578\times10^6\dot{\beta}$$
$$+2\times90\times10^3\left[.2818\,\alpha + .01325\,\dot{\alpha} + .1659\,\ddot{\phi} + .03375\,\ddot{\beta} - {}^{*}.3568\,\dot{\phi}\right.$$
$$\left. - {}^{*}.7255\,\dot{\beta} + 9.448\left(k_e\dot{\beta} + K_e\beta\right)\right] + 2\times112.56\times10^3\left(k_a\dot{\beta} + K_a\beta\right)$$

Re-group terms and introducing D operator.

$$\left(9.056D^2 + 8.065D\right)\phi + \left[1.516D^2 + \left(1.706 - 1.701k_e - .2251k_a\right)D - \left(1.701K_e + .2251K_a\right)\right]\beta$$
$$-\left(.2039D + 23.08\right)\alpha = L_o\times10^{-6} + 1.493\left(2\dot{\phi}\dot{\beta} + \dot{\beta}^2\right)\beta \qquad (22)$$

* It should be noted that effect of fighter wing on $\dot{\phi}$ + $\dot{\beta}$ are included in $L_{\dot{\phi}}$ and $L_{\dot{\beta}}$.

Fighter Flap Equation

$$m\left[k_x^2 + r(R+r)\right]\ddot{\phi} + m(k_x^2 + r^2)\ddot{\beta} + mRr\dot{\phi}^2\beta$$
$$= L_1 + \Sigma L_w a_w + r\Sigma L_t + L_{\delta_a}\delta_a$$

$$453.4(\overline{5.61}^2 + 18.3\times 90)\ddot{\phi} + 453.4(\overline{5.61}^2 + \overline{18.3}^2)\ddot{\beta} + 453.4\times 7.17\times 18.3\dot{\phi}^2\beta$$
$$= L_1 + 119.4\times 17.95\times 10^3\alpha - 21.85\times 17.62\times 10^3\dot{\phi}$$
$$-4.871\times 21.6\times 10^3\dot{\beta} + 18.3\times 10^3\left[8.784\alpha + 1.1325\dot{\alpha}\right.$$
$$+ .1659\ddot{\phi} + .03375\ddot{\beta} - 3.568\dot{\phi} - .7255\dot{\beta} + 9.448 k_e\dot{\beta}$$
$$\left. + 9.448 K_e\beta\right] + 112.56\times 10^3(k_a\dot{\beta} + K_a\beta)$$

Simplifying,

$$(7.58D^2 + 4.503D)\phi + \left[1.655D^2 + (1.185 - 1.729k_e - 1.1256k_a)D\right.$$
$$\left. - (1.729K_e + 1.1256K_a)\right]\beta - (.2071D + 23.037)\alpha$$
$$= L_1\times 10^{-5} - 7.476\dot{\phi}^2\beta \quad \text{- - - - - - - - - - - - - - - -} \quad (23)$$

Fighter-Pitch Equation

For c = o, Eq. (11) reduces to

$$-I_y\ddot{\alpha} + L_w d - L_t l_t + M_F = M_1$$

$$-18.595\ddot{\alpha} + .2217(119.4\alpha - 21.85\dot{\phi} - 4.871\dot{\beta})$$
$$-18.14\left[8.784\alpha + 1.1325\dot{\alpha} + .1659\ddot{\phi} + .03375\ddot{\beta}\right.$$
$$\left. - 3.568\dot{\phi} - .7255\dot{\beta} + 9.448(k_e\dot{\beta} + K_e\beta)\right]$$
$$+ 79.86\alpha = M_1\times 10^{-3}$$

Simplifying,

$$(-3.0094D^2 + 59.88D)\phi + [-.612D^2 + (12.081 - 171.39k_e)D + 171.39K_e]\beta$$
$$-(18.595D^2 + 20.544D + 53.01)\alpha = M_1 \times 10^{-3} \quad \text{------} (24)$$

Characteristic Equation

Neglecting non-linear terms, the characteristic equation is

$$D\begin{vmatrix} -3.0094D + 59.88 & -.612D^2 + (12.081 - 171.39k_e)D - 171.39K_e & -18.595D^2 - 20.544D - 53.01 \\ 7.58D + 4.503 & 1.655D^2 + (1.185 - 1.729k_e - 1.1256k_a)D - (1.729K_e + 1.1256K_a) & -.2071D - 23.037 \\ 9.056D + 8.065 & 1.516D^2 + (1.076 - 1.701k_e - .2251k_a)D - (1.701K_e + .2251K_a) & -.2039D - 23.08 \end{vmatrix} = 0 \quad \text{--------} (25)$$

Expanding Eq. (25), and collecting coefficients in like powers in D, (p. 20)

$$(26) \text{----} \begin{cases} a_0 = 65.004 \\ a_1 = 241.169 - 51.464k_e - 157.82k_a \\ a_2 = 485.28 - 116.9k_e - 324.87k_a - 51.464K_e - 157.82K_a \\ a_3 = 23.573 + 5637.4k_e - 667.2k_a - 116.9K_e - 324.87K_a \\ a_4 = -892.2 + 13750k_e + 817.08k_a + 5637.4K_e - 667.2K_a \\ a_5 = 13750K_e + 817.08K_a \end{cases}$$

(b) Effect of C.G. Location

The effect of changing the c.g. location of the fighter relative to the hinge point is to provide inertia moments in producing pitch rotation. This effect on stability may be studied by considering the terms in Fighter-pitch equation containing "c".

$$\left\{ mc(R+r)\ddot{\phi} + mcr\ddot{\beta} - c\sum(L_w + L_t) \right\} \times 10^{-3}$$

$$= C\left\{ 453.4 \times 90\,\ddot{\phi} + 453.4 \times 18.3\,\ddot{\beta} - \left[128.18\,\alpha - 25.418\,\dot{\phi} - 5.5965\,\dot{\beta} \right.\right.$$

$$\left.\left. + 1.1325\,\dot{\alpha} + .1659\,\ddot{\phi} + .03375\,\ddot{\beta} + 9.448\,(k_e\dot{\beta} + K_e\beta) \right] \right.$$

$$- C\left\{ (40.64\,D^2 + 25.418\,D)\phi + \left[8.26345\,D^2 + (5.5965 - 9.448\,k_e)D - 9.448\,K_e \right]\beta \right.$$

$$\left. - (1.1325\,D + 128.18)\,\alpha \right\} \text{ - - - - - - - - - - - - - - } (27)$$

The characteristic equation (25) is, then, modified by adding the following determinant to it

$$\begin{vmatrix} 40.64D + 25.418 & 8.26345\,D^2 + (5.5965 - 9.448\,k_e)D - 9.448\,K_e & -(1.1325\,D + 128.18) \\ 7.58D + 4.503 & 1.655\,D^2 + (1.185 - 1.729\,k_e - 1.1256\,k_a)D - (1.729\,K_e + 1.1256\,K_a) & -(0.2071\,D + 23.037) \\ 9.056D + 8.065 & 1.516\,D^2 + (1.076 - 1.701\,k_e - .2251\,k_a)D - (1.701\,K_e + .2251\,K_a) & -(0.2039\,D + 23.08) \end{vmatrix} CD = 0 \text{ - - - - - - - - - } (28)$$

Expanding, and collecting the increments in coefficients,

$$
(29)\ \text{------}\ \left\{
\begin{aligned}
\Delta a_0 &= 0 \\
\Delta a_1 &= .2784\,C \\
\Delta a_2 &= (37.713 - 2.183\,k_a)\,c \\
\Delta a_3 &= C\,(114.256 - 6.898\,k_e - 247.7\,k_a - 2.183\,K_a) \\
\Delta a_4 &= C\,(80.604 - 13.809\,k_e - 505.42\,k_a - 6.898K_e \\
&\qquad - 247.7\,K_a) \\
\Delta a_5 &= C\,(-13.809\,K_e - 505.42\,K_a)
\end{aligned}
\right.
$$

(c) Effect of Auto-Pilot in Bomber

The ailerons of the bomber may be assumed to deflect in proportion to ϕ and $\dot{\phi}$ such that stabilizing influence is imposed on the coupled flight. The increment in the coefficients of the characteristic equation may be found by introducing two terms in the bomber-roll equation, namely, $k_\phi D + K_\phi$. The determinant

$$
(k_\phi D + K_\phi)\begin{vmatrix} -.612D^2 + (12.081 - 171.39k_e)D - 171.39K_e & -(18.595D^2 + 20.544D + 53.01) \\ 1.6549D^2 + (1.1849 - 1.729k_e - 1.1256k_a)D - (1.729K_e + 1.1256K_a) & -(.2073D + 23.04) \end{vmatrix}
$$

$$
+ (k_\phi D + K_\phi)C\begin{vmatrix} 8.2634D^2 + (5.597 - 9.448k_e)D - 9.448K_e & -(1.1325D + 128.18) \\ 1.6549D^2 + (1.1849 - 1.729k_e - 1.1256k_a)D - (1.729K_e + 1.1256K_a) & -(.2073D + 23.04) \end{vmatrix} \quad \text{--------}\ (30)
$$

gives the following increments in the coefficient of the characteristic equations:

$$\Delta a_0 = 0$$

$$\Delta a_1 = 30.773\ k_\phi$$

$$\Delta a_2 = \left[(56.16 + .163\ c) - 32.15\ k_e - 20.931\ k_a\right] k_\phi + 30.773\ K_\phi$$

$$\Delta a_3 = \left[(123.67 + 22\ c) - (23.124 + 1.275\ c)k_a - 32.15\ K_e - 20.931\ K_a\right] k_\phi$$
$$+ \left[(56.16 + .163\ c) - 32.15\ k_e - 20.931\ k_a\right] K_\phi$$

$$\Delta a_4 = \left[(-215.53 + 23\ c) + (3857.1 - 3.9\ c)k_e - (59.67 + 144.0c\ )k_a\right.$$
$$\left. - (23.124 + 1.275\ c)K_a\right] k_\phi + \left[(123.67 + 22\ c) - (23.124 + 1.275\ c)k_a\right.$$
$$\left. - 32.15\ K_e - 20.931\ K_a\right] K_\phi$$

$$\Delta a_5 = \left[(3857.1 - 3.9\ c)K_e - (59.67 + 144\ c)\ K_a\right] k_\phi$$
$$+ \left[(-215.53 + 23\ c) + (3857.1 - 3.9\ c)k_e - (59.67 + 144\ c)\ k_a\right.$$
$$\left. - (23.124 + 1.275\ c)K_a\right] \cdot K_\phi$$

$$\Delta a_6 = \left[(3857.1 - 3.9c)K_e - (59.67 + 144\ c)K_a\right] K_\phi$$

The coefficients given in Eq. (26), (29) and (31) may be combined. After dividing them by $a_o = 65.004$, these characteristic equation coefficients become:

$$a_0 = 1.0$$

$$a_1 = (3.7101 + .00428\ c) - .7917\ k_e - 2.4279\ k_a + .4734\ k_\phi$$

$$a_2 = (7.4654 + .5802\ c) - 1.7984\ k_e - (4.9976 + .03358\ c)\ k_a$$
$$- .7917\ K_e - 2.4279\ K_a + \left[(.864 + .0025\ c) - .4946\ k_e\right.$$
$$\left. - .322\ k_a\right] k_\phi + .4734\ K_\phi$$

$$a_3 = (.3626 + 1.7577\ c) + (86.724 - .1061\ c)k_e - (10.264 + 3.8109c)k_a$$
$$- 1.7984\ K_e - (4.9976 + .03358\ c)K_a + \left[(1.903 + .3384\ c)\right.$$
$$\left. - (.3557 + .0196c)k_a - .4946\ K_e - .322\ K_a\right] k_\phi$$
$$+ \left[.864 + .0025c - .4946k_e - .322k_a\right] K_\phi$$

$$a_4 = (-13.7252 + 1.240c) + (211.52 - .2124c)k_e + (12.5697 - 7.775c)k_a$$
$$+ (86.7243 - .1061c)K_e - (10.264 + 3.8109c)K_a +$$
$$\Big[(-3.3157 + .3539c) + (59.34 - .06c)k_e - (.918 + 2.215c)k_a$$
$$- (.3557 + .0196c)K_a\Big] k_\phi + \Big[(1.903 + .3384c) - (.3557 + .0196c)k_a$$
$$- .4946K_e - .322\,K_a\Big] K_\phi$$

$$a_5 = (211.52 - .2124c)K_e + (12.5697 - 7.775c)K_a +$$
$$+ \Big[(59.34 - .06c)K_e - (.918 + 2.215c)K_a\Big] k_\phi +$$
$$+ \Big[(-3.3157 + .3539c) + (59.34 - .06c)k_e - (.918 + 2.215c)k_a$$
$$- (.3557 + .0196c)K_a\Big] K_\phi$$

$$a_6 = \Big[(59.34 - .06c)K_e - (918 + 2.215c)K_a\Big] K_\phi$$

In the testing of the stability criteria in accordance with Eq. (19), the parameters:

Auto-Pilot of Bomber	=	k_ϕ	,	K_ϕ
Auto-Pilot of Fighter	=	k_e	,	K_e
		k_a	,	K_a
L.G. Location	=		c	

must be studied

2. Pitch Stability

The equations of motion (13), (14) and (15) may be written in such a way that the terms containing A and c are separated from those which do not. Equation (13) becomes

$$\left\{ (I_{YY} D^2 - M_{\dot{\theta}} D - M_{\theta}) \theta \right\} + 2 \left\{ m A^2 \ddot{\theta} - m A r \ddot{\beta} + m A C \ddot{\alpha} + A \sum (L_w - L_t) \right\}$$

$$= M_o$$

Substituting,

$$(10^6 D^2 + .71337 \times 10^6 D + .93478)\theta + 2\left\{ 453.4 A^2 \ddot{\theta} - 453.4 \times 183 A \ddot{\beta} \right.$$
$$+ 453.4 AC \ddot{\alpha} + A \times 10^3 [128.18 \alpha + 1.1325 \dot{\alpha} + .03375 \ddot{\beta}$$
$$\left. - 5.5965 \dot{\beta} + 22.8 \theta + 9.448 (k_e \dot{\beta} + K_e \beta)] \right\} = M_o$$

$$\left\{ (D^2 + .71337 D + .93478)\theta \right\} + \left\{ (.9068 \times 10^{-3} A^2 D^2 + .0456 A)\theta \right.$$
$$+ A[-.01652 D^2 - (.011193 - .0189 k_e) D + .0189 K_e]\beta$$
$$\left. + A[.9068 \times 10^{-3} C D^2 + 2.265 \times 10^{-3} D + .25637]\alpha \right\}$$
$$= M_o \times 10^{-6} \qquad \text{------------------} (32)$$

Let

$$a_1 = D^2 + 0.71337\,D + .93478$$

$$a_2 = 0$$

$$a_3 = 0$$

$$a_1' = .9068\times10^{-3}\,A^2D^2 + .0456\,A$$

$$a_2' = A\left[-.01652D^2 + (.0189k_e - .011193)D + .0189K_e\right]$$

$$a_3' = .9068\times10^{-3}\,AC\,D^2 + A(.002265D + .25637)$$

The fighter-flap equation (14) may be rearranged to give:

$$\left\{m(k_x^2 + r^2)\ddot{\beta} - \Sigma L_w a_w - r\Sigma L_t + L_{\delta_a}\delta_a\right\} - \left\{mAr\ddot{\theta} + mcr\ddot{\alpha}\right\} = L_2$$

Substituting,

$$\left\{453.4(\overline{5.61}^2 + \overline{18.3}^2)\ddot{\beta} - (119.4\times17.95\,\alpha - 4.871\times21.6\,\dot{\beta} + 22.8\times9.67\,\theta)\times10^3 - 18.3\left[8.784\,\alpha + 1.1325\,\dot{\alpha} + .03375\,\ddot{\beta} - .7255\,\dot{\beta} + 9.448(k_e\dot{\beta} + K_e\beta)\right] + 112.56\times10^3(k_a\dot{\beta} + K_a\beta)\right.$$
$$= L_2$$

$$\left\{-4.1724\,\theta + \left[1.6549D^2 + (1.1849 - 1\text{-}729k_e - 1.1256k_a)D - (1.729K_e + 1.1256K_a)\right]\beta + \left[-.20725D - 23.04\right]\alpha\right\} + \left\{-.08297AD^2\,\theta - .08297\,CD^2\alpha\right\} = L_2\times10^{-5}$$

---------- (33)

Let

$$b_1 = -\,4.1724$$

$$b_1' = -\,.082972\ A\ D^2$$

$$b_2 = 1.6549D^2 + (1.1849 - 1.729k_e - 1.1256k_a)D$$
$$-\,(1.729K_e + 1.1256K_a)$$

$$b_2' = 0$$

$$b_3 = -\,(.20725\ D + 23.04)$$

$$b_3' = -\,.082972\ C\ D^2$$

The Fighter-Pitch Equation (15), after rearranging, becomes

$$\left\{I_y\ddot{\alpha} - M_{F_\alpha}\alpha - d\Sigma L_w + l_t\Sigma L_t\right\} + \left\{mAC\ddot{\theta} - mCr\ddot{\beta} + mC^2\ddot{\alpha} + C\Sigma(L_w + L_t)\right\} = M_2$$

Substituting,

$$\{18.595\ddot{\alpha} - 79.86\alpha - .2217(119.4\alpha - 4.871\dot{\beta} + 22.8\theta) + 18.14[8.784\alpha$$
$$+ 1.1325\dot{\alpha} + .03375 - .7255\dot{\beta} + 9.448(k_e\dot{\beta} + K_e\beta)]\}$$
$$+\{.4534\,AC\ddot{\theta} - .4534 \times 18.3C\ddot{\beta} + .4534\,C^2\ddot{\alpha} + C[128.18\alpha$$
$$+ 1.1325\dot{\alpha} + 22.8\theta + .03375\ddot{\beta} - 5.5965\dot{\beta} + 9.448k_e\beta$$
$$+ 9.448K_e\beta]\} = M_2 \times 10^{-3}$$

$$\{-5.0548\,\theta + [.612 D^2 + (171.39k_e - 12.08)D + 171.39K_e]\beta + [18.595 D^2 + 20.54 D + 53.01]\alpha\} + \{(.4534\, AC D^2 + 22.8 C)\,\theta + [-8.2635 C D^2 + (9.448k_e - 5.5965)C \cdot D + 9.448 K_e C]\beta + [.4534 C^2 D^2 + 128.18 C]\alpha\} = M_2 \times 10^{-3} \quad \text{------------- (34)}$$

Let

$$c_1 = -5.0548$$

$$c_1' = .4534\ AC\ D^2 + 22.8\ C$$

$$c_2 = .612\ D^2 + (171.39k_e - 12.08)D + 171.39K_e$$

$$c_2' = C\left[-8.2635\ D^2 + (9.448k_e - 5.5965)D + 9.448K_e\right]$$

$$c_3 = 18.595\ D^2 + 20.54D + 53.01$$

$$c_3' = .4534\ C^2 D^2 + C(1.1325\ D + 128.18)$$

The characteristic equation is

$$\begin{vmatrix} a_1 + a_1' & a_2 + a_2' & a_3 + a_3' \\ b_1 + b_1' & b_2 + b_2' & b_3 + b_3' \\ c_1 + c_1' & c_2 + c_2' & c_3 + c_3' \end{vmatrix} = \Delta = 0 \quad \text{--- (35)}$$

This determinant may be expanded by minor as:

$$\Delta = (a_1 + a_1')M_1 - (b_1 + b_1')M_2 + (c_1 + c_1')M_3$$

where

$$M_1 = b_2 c_3 - c_2 b_3 + b_2 c_3' - b_3' c_2 - b_3 c_2' - b_3' c_2'$$

$$M_2 = -a_3' c_2 + a_2' c_3 + a_2' c_3' - a_3' c_2'$$

$$M_3 = -a_3' b_2 + a_2' b_3 + a_2' b_3'$$

After expanding and term-collecting, the following constants for the polynominal is obtained:

$$a_0 = 30.773 + .05078c + .06469c^2 + .0024A^2$$

$$a_1 = (78.105 - .8045c + .11895c^2 + .00517A^2)$$
$$-(32.151 - 14.22c)k_e - (20.931 + 5103c^2 + .01898A^2)k_a$$

$$a_2 = (192.49 + 21.37c + .1174c^2 + .1201A + .0096A^2 + .000657AC)$$
$$-(22.929 - 10.144c)k_e - (38.05 + 1.2747c + .364c^2 + .02096A^2)k_a$$
$$-(32.151 - 14.22c)K_e - (20.931 + .5103c^2 + .1898A^2)K_a$$

$$a_3 = (-74.8 + 37.8c + .06805c^2 + .272A + .01233A^2 + .000733AC)$$
$$+(3827.1 + 9.343c - .0649A)k_e - (95.727 + 145.19c + .477c^2)$$
$$+(95.727 + 145.19c + .477c^2 + .9545A + .05411A + .006061\ AC)k_a$$
$$-(22.929 - 10.144c)K_e - (38.05 + 1.2747c + .364c^2 + .02096A)K_a$$

$$a_4 = (-38.15 + 36.862c + .7053A + .001AC) + (2751.6 - 2.818c + .00032A)k_e$$
$$-(64.177 + 104.13c + 1.0672A)k_a + (3827.1 + 9.343c - .0649)K_e$$
$$-(95.727 + 145.19c + .477c + .9545A + .05411A + .006061AC)K_a$$

$$a_5 = (-201.45 + 21.453c + .8506A + .0007AC) + (3605.6 - 3.6924c$$
$$-3.3A - .0032AC)k_e - (55.776 + 134.87c + 4.1798A)k_a$$
$$+(2751.6 - 2.818c + .00032A)K_e - (64.177 + 104.13c + 1.0672A)K_a$$

$$a_6 = (3605.6 - 3.6924c - 3.3A - .0032AC)K_e - (55.776 + 134.87c$$
$$+ 4.1798A)K_a$$

Effect of Auto-Pilot in Bomber

It will be shown that stability of the coupled flight is difficult to obtain if the bomber were left uncontrolled. The effect of the auto-pilot control in the bomber should be more effective since the controls on the bomber are more powerful than those of the fighters. The introduction of auto-pilot in the bomber modifies the Bomber-pitch equation by adding two terms

$$(k_\theta D + K_\theta)\theta$$

where k_θ and K_θ are proportional, control moments responding to the rate and the displacement changes of the pitching angle of the bomber. The characteristic equation (35) is, then, modified by adding

$$(k_\theta D + K_\theta) M_1$$

where M_1 is the first minor in the expansion of the determinant. The incremental coefficients are:

$$\Delta a_0 = 0$$

$$\Delta a_1 = (30.773 + .05078c + .06469c^2)k_\theta$$

$$\Delta a_2 = \Big[(56.152 - 32.151k_\theta - 20.931k_a) + (14.22k_\theta - .8407)c + (.0728 - .5103k_a)c^2\Big]k_\theta + (30.773 + .05078c + .06469c^2)K_\theta$$

$$\Delta a_3 = \Big[(123.66 + .007k_\theta - 32.151K_\theta - 23.12k_a - 20.931K_a) + (21.92 + 14.22K_\theta - 1.2747k_a)c - .5103c^2K_a\Big]k_\theta + \Big[(56.152 - 32.151k_\theta - 20.931k_a) + (14.22k_\theta - .8407)c + (.0728 - .5103k_a)c^2\Big]K_\theta$$

$$\Delta a_4 = \Big[(-215.51 + 3857.1k_e - 59.668k_a + .007K_e - 23.12K_a) + (22.95 - 3.95k_e - 144.28k_a - 1.2747K_a)C\Big]k_\theta + \Big[(123.66 + .007k_e - 32.151K_e - 23.12k_a - 20.931K_a) + (21.92 - 14.22K_e - 1.2747k_a)C - .5103c^2K_a\Big]K_\theta$$

$$\Delta a_5 = \Big[(3857.2K_e - 59.668K_a) - (3.95K_e + 144.28K_a)C\Big]k_\theta + \Big[(-215.51 + 3857.1k_e - 59.668k_a + .007K_e - 23.12K_a) + (22.95 - 3.95k_e - 144.28k_a - 1.2747K_a)C\Big]K_\theta$$

$$\Delta a_6 = \Big[(3857.1K_e - 59.668K_a) - (3.95K_e - 144.28K_a)C\Big]K_\theta$$

In the testing of the stability criteria in accordance with Eq. (19), the following parameters are considered:

Auto-Pilot of Bomber:	k_θ	K_θ
Auto-Pilot of Fighters:	k_e	K_e
	k_a	K_a
C.G. Locations:	A and	C

Discussion of Results

The criteria for the stability in roll as well as in pitch were investigated simultaneously due to the fact that the same auto-pilot in the fighter must work for both disturbances. Because of the number of parameters involved, a somewhat systematic analysis was made. Values of the rate controls in the fighter auto-pilot, k_e and k_a were first assumed. It is, then, possible to plot a region of stable flight using the displacement controls in the fighter's auto-pilot, K_e and K_a as coordinates. The criteria $a_1 > 0$, $a_1a_2 - a_0a_3 > 0$, $a_1a_4 - a_0a_5 > 0$ are equations of straight lines (linear in K_e and K_a), and $a_3(a_1a_2 - a_0a_3) - a_1(a_1a_4 - a_0a_5) > 0$ is the equation of an ellipse or a hyperbola. The other criteria are higher-degree curves generally not important.

The order of magnitude of these parameters should be estimated. That for the fighter elevator movement, i.e., k_e and K_e may be estimated from the elevator angle required to produce 1_g acceleration in pull-ups.

$$\frac{\Delta\delta_e}{\Delta g} = \frac{l_w}{q}\frac{1}{C_{L_{\delta_e}}}\frac{1}{V_t}\left(\frac{dC_M}{dC_L}\right)_{FREE} + \frac{\sigma}{2}\rho_0 g\, l_t\left(\frac{C_{H_\alpha}}{C_{H_{\delta_e}}} - \frac{1}{\tau}\right)$$

where l_w = wing loading = W/S = $14,600/260$ = 56.2 lbs/sq. ft.

q = 103.5 lbs./sq. ft.

V_t = tail volume = .454

l_t = 18.14 ft.

$C_{L_{t_{\delta_e}}}$ = .033 per degree

τ = .538 = $\partial\alpha/\partial\delta_e$

$$(d C_M / d C_L) \text{ free} = -.03 \text{ assuming 3\% stick-free margin}$$

$$C_{H_\alpha} = -.001 \quad \text{(Ref.3)} \quad \text{(per degree)}$$

$$C_{H_{\delta_e}} = -.0052 \quad \text{(Ref. 3)} \text{(per degree)}$$

$$\sigma = .448 \text{ @ } 25{,}000 \text{ ft.}$$

$$\therefore \quad \frac{\Delta \delta_e}{\Delta g} = -1.26 \text{ degrees per 'g'}$$

Since no high-accelerated flight is expected in 'α' stable flight, the values of k_e and K_e should be less than unity.

$$k_e = \frac{\Delta \delta_e}{\Delta \dot{\beta}} < 1.0$$

$$K_e = \frac{\Delta \delta_e}{\Delta \beta} < 1.0$$

The largest values for k_a and K_a, the aileron parameter is in the order of 3 since the aileron deflection is limited to 18° in which case a flipping angle of $\beta = \pm 7^\circ$ may be permitted.

Referring to Eq. (30) and (37), the parameters in the bomber auto-pilot are defined rather loosely. Considering the number involved in the damping and spring-restoring coefficients in θ - equation, the k_θ and K_θ should be in the order of magnitude of unity, i.e., the controls should be able to produce coefficients in the same order of magnitude as the coefficient of the system in $\dot{\theta}$ and θ without controls. For the same reason, k_ϕ and K_ϕ should be in the order of magnitude of 4.

The case of no controls in the bomber was investigated first. For this case, it can readily be shown ath the $k_e < .15$ and $k_a > 0$ are required for stability in both pitch and roll. Table I and Figs. 5 to 10 present the cases investigated. The areas hatched in one direction are zones of roll stability while those hatched in opposite directions are zones of pitch stability. Only the values of K_e and K_a in the over-lapped areas are acceptable combinations. It is seen that the stability of the system is very sensitive to the control parameters. Within the range of c.g. locations likely in the actual system, viz., $c = 0$ to 1 and $A = 0$ to 3.25, the effect of c.g. locations on the stability of the system is relatively small.

Figs. 11 and 12 show the stability boundaries for a given auto-pilot in the fighters, viz., $k_e = .1$ and $K_e = .03$ using the bomber auto-pilot parameters k_ϕ, K_ϕ, k_θ and K_θ as coordinates. Apparently, any combination of positive k_ϕ and K_ϕ, and k_θ and K_θ would improve the stability of the system. At this point, it was believed that the bomber should not be left uncontrolled. To verify this contention, values of $k_\phi = K_\phi = .4$, and $k_\theta = K_\theta = 1$ were arbitrarily assigned. The resulted stability boundaries for the pitch and roll disturbances were shown in Fig. 13 and Fig. 14 for the case considered. It is evident that the system can be readily made stable if judicious auto-pilot or manual controls were provided in the bomber.

3. Yaw Stability

$$\left[I_z + 2m\left(k_z^2 + \overline{R+r}^2\right)\right]\ddot{\psi} + 2m\left[k_z^2 + r(R+r)\right]\ddot{\gamma} - 2mRr\left(2\dot{\psi}\dot{\gamma} + \dot{\gamma}^2\right)$$

$$= N_0 + \sum N_{\dot{\psi}}\dot{\psi} + \sum N_{\psi}\psi + 2N_{\dot{\psi}_{F-84}}\dot{\gamma} + 2N_{\psi_{F-84}}\gamma$$

$$\left[2.712\times10^6 + 2\times453.4\left(8.36^2 + 90^2\right)\right]\ddot{\psi} + 2\times453.4\left[8.36^2 + 18.3\times90\right]\ddot{\gamma}$$

$$-2\times453.4\times71.7\times18.3\left(2\dot{\psi}\dot{\gamma} + \dot{\gamma}^2\right) = N_0 - .532\times10^6\dot{\psi} - 4.725\times10^6\psi$$

$$-.01392\times10^6\dot{\gamma} - .2596\times10^6\gamma.$$

$$\left(10.12D^2 + .532D + 4.725\right)\psi + \left(1.557D^2 + .01392D + .2596\right)\gamma$$

$$= N_0\times10^{-6} - 1.19\left(2\dot{\psi}\dot{\gamma} + \dot{\gamma}^2\right) \quad ------ \quad (38)$$

and,

$$\left(m\left[k_z^2 + r\,\overline{R+r}\right]\right)\ddot{\psi} + m\left(k_z^2 + r^2\right)\ddot{\gamma} + mRr\dot{\psi}^2\gamma$$

$$= N_1 + N_{\dot{\psi}_{F-84}}\left(\dot{\psi} + \dot{\gamma}\right) + N_{\psi_{F-84}}\left(\psi + \gamma\right)$$

$$\left[453.4\left(8.36^2 + 18.3\times90\right)\right]\ddot{\psi} + 453.4\left(8.36^2 + 18.3^2\right)\ddot{\gamma}$$

$$+ 453.4\times71.7\times18.3\,\dot{\psi}^2\gamma = N_1 - .00696\times10^6\left(\dot{\psi} + \dot{\gamma}\right)$$

$$- .1298\times10^6\left(\psi + \gamma\right)$$

$$\left(.7785D^2 + .00696D + .1298\right)\psi + \left(.1835D^2 + .00696D + .1298\right)\gamma$$

$$= N_1\times10^{-6} - .5479\left(2\dot{\psi}\dot{\gamma} + \dot{\gamma}^2\right) \quad ------ \quad (39)$$

The characteristic polynomial for this case is

$$\begin{vmatrix} 10.12 D^2 + .532 D + 4.725 & 1.557 D^2 + .01392 D + .2596 \\ .7785 D^2 + .00696 D + .1298 & .1835 D^2 + .00696 D + .1298 \end{vmatrix}$$

$$= .645 D^4 + .14638 D^3 + 1.78 D^2 + .09971 D + .5796 = 0$$

or,

$$D^4 + .2269 D^3 + 2.760 D^2 + .1546 D + .8986 = 0$$

The stability criteria for this case occurred in the classical treatment of airplane stability and is known as the Roth Discriminant.

$$a_1 a_2 a_3 - a_3^2 - a_1^2 a_4 > 0$$

$$.2269 \times 2.76 \times .1546 - \overline{.1546}^2 - \overline{.2269}^2 \times .8986$$

$$= .8985 > 0$$

It is seen that the yaw stability with the degrees of freedom assumed is positive. It appears that rudder correction in the auto-pilot in the fighter need not be used.

Case B - Two-Joint Attachment

1. Roll Stability

Degree of Freedom:

Bomber Roll = ϕ

Fighter Flap = β

The equations of motions derived in Case A hold good in this case also. Since the fighter is locked such that pitching of the fighter relative to the bomber is not possible, the aerodynamic forces on the fighter wing and tail as given in Eq. I and III have to be modified.

Bomber-Roll ϕ - Equation

From Eq. 7,

$$\left\{ I_0 + 2m\left[k_x^2 + \overline{R+r}^2\right]\right\}\ddot{\phi} + 2m\left[k_x'^2 + r(R+r)\right]\ddot{\beta} - 2Rr(2\dot{\beta}\dot{\phi} + \dot{\beta}^2)\beta$$
$$= L_0 + L_{\dot{\phi}}\dot{\phi} + L_{\dot{\beta}}\dot{\beta} + L_\beta \beta$$

where L_β is the rolling moment provided by the symmetric deflection of the flaps or ailerons of the fighters. For the condition considered,

$$L_\beta = \left[C_{L_\alpha} \cdot \alpha_{\delta_f} \cdot q \cdot S\right] 2(R+r) K$$

where

$$K = \frac{\Delta \delta_f}{\Delta \beta}$$

For the purpose of study, it is assumed that the flap is full-span, and $\alpha_{\delta_f} = .5$,

$$\therefore \; L_\beta = (4.439 \times .5 \times 103.5 \times 260) \times 2 \times 90 K$$

$$= 59.73 \times 10^3 K \times 2(R+r) = 10.752 \times 10^6 K \text{ - FT-LB.}$$

$$\therefore \; [1.712 \times 10^3 + 2 \times 453.4(5.61^2 + 90^2)]\ddot{\phi} + 2 \times 453.4[5.61^2 + 18.3 \times 90]\ddot{\beta}$$

$$-2 \times 453.4 \times 71.7 \times 18.3(2\dot{\phi}\dot{\beta} + \dot{\beta}^2)\beta$$

$$= L_o - 7.423 \times 10^6 \dot{\phi} - .94578 \times 10^6 \dot{\beta} + 10.752 \times 10^6 K\beta$$

$$(9.093 D^2 + 7.423 D)\phi + (1.524 D^2 + .94578 D - 10.752K)\beta$$

$$= L_o \times 10^{-6} + 1.495(2\dot{\phi}\dot{\beta} + \dot{\beta}^2)\beta \quad \text{------------} \quad (40)$$

<u>Fighter-Flap</u> β- equation

From Equation 8,

$$m[k_x^2 + r(R+r)]\ddot{\phi} + m(k_x^2 + r^2)\ddot{\beta} + mRr\dot{\phi}^2\beta$$

$$= L_1 + \Sigma L_w a_w + r \Sigma L_t$$

where

$$L_w = L_{w_{\dot{\beta}}}\dot{\beta} + L_{w_{\dot{\phi}}}\dot{\phi} + L_{\delta_f} \cdot \delta_f \qquad (\text{see Eq. I.})$$

$$= -21.85 \times 10^3 \dot{\phi} - 4.871 \times 10^3 \dot{\beta} + 59.73 \times 10^3 K\beta$$

$$L_t = 165\ddot{\phi} + 33.5\ddot{\beta} - 3.546 \times 10^3 \dot{\phi} - 720.9\dot{\beta} \qquad (\text{see Eq. III})$$

$$453.4[5.61^2+18.3\times 90]\ddot{\phi} + 453.4[5.61^2+18.3^2]\ddot{\beta} - 453.4\times 71.7\times 18.3\,\dot{\phi}^2\beta$$
$$=[-21.85\times 16.95\times 10^3\dot{\phi} - 4.871\times 21.6\times 10^3\dot{\beta} + 59.73\times 17.95\times 10^3 K\beta]$$
$$+18.3\,(165\ddot{\phi} + 33.5\ddot{\beta} - 3.546\times 10^3\dot{\phi} - 720.9\dot{\beta}) + L_1$$

$$(7.576 D^2 + 4.353 D)\phi + (1.657 D^2 + 1.184 D - 10.72 K)\beta$$
$$= L_1\times 10^{-5} - 7.476\,\dot{\phi}^2\beta \qquad \text{-------------------- (41)}$$

The characteristic equation becomes

$$\begin{vmatrix} 7.576D^2 + 4.353D & 1.657D^2 + 1.184D - 10.72K \\ 9.093D^2 + 7.423D & 1.524D^2 + .94574D - 10.752K \end{vmatrix} = 0$$

Expanding,

$$D\left[D^3 + 2.632D^2 + (1.327 - 4.554K)D - 9.309K\right] = 0$$

Using the Routh's discriminant,

$$2.632\,(1.327 - 4.554K) - 9.309K > 0$$

$$3.493 - 2.677K > 0$$

$$\therefore\ K < 1.305$$

And

$$K < 0$$

Since the constant term 9.309K requires that K be negative, the system is stable for all negative values of K.

$$\boxed{\text{Select } K = -1}$$

$$D(D^3 + 2.632D^2 + 5.881D + 9.309) = 0$$

The roots of this polynominal are:

$D = 0$

$D = -2.0112$

$D = -.3104 \pm 2.13\, i$

Circular frequency, $\omega = 2.13$

Period, $P = 2\pi/\omega = 2.95$ sec.

2. Pitch Stability

Degrees of Freedom:

Bomber Pitch : θ

Fighter Flap : β

This equation of motion may be written down from Eq. (13), (14) and (15) by locking the pitch motion of the fighter; i.e. $\alpha = \theta$.

Pitch Equation of the Combination

Equation (13) plus 2 x Eq. (15), and remembering $\alpha = \theta$, gives

$$\left\{I_{YY} + 2\left[I_y + m(c+A)^2\right]\right\}\ddot{\theta} - 2mr(C+A)\ddot{\beta} + 2m(C+A)\left[r\dot{\beta}^2\beta - (C+A)\dot{\theta}^2\theta\right]$$

$$= M_o + M_\theta \theta + M_{\dot{\theta}}\dot{\theta} + 2(d-c-A)\Sigma L_w - 2(l_t + C + A)\Sigma L_t + 2M_{f\alpha}\cdot\theta$$

Substituting values for the condition under consideration,

$$
\begin{aligned}
&\left\{1000\times10^{3} + 2\left[18.595 + 453.4\,(3.25+.5912)^{2}\right]\right\}\ddot{\theta} - 2\times453.4\times18.3\,(3.25+.5912)\,\ddot{\beta} \\
&\quad + 2\times453.4\,(3.25+.5912)\left[18.3\,\dot{\beta}^{2}\beta - (3.25+.5912)\,\dot{\theta}^{2}\theta\right] \\
&= M_0 - .93474\times10^{6}\,\theta - .71337\times10^{6}\,\dot{\theta} + 2\,(22.17 - 3.25 - .5912)\left[-4.871\times10^{3}\,\dot{\beta}\right. \\
&\quad \left. + 142.24\times10^{3}\,\theta + 59.73\times10^{3}\,K\beta\right] - 2\,(18.14 + 3.25 + .5912)\left[8.784\times10^{3}\,\theta\right. \\
&\quad \left. + 1.1325\times10^{3}\,\dot{\theta} + .03375\times10^{3}\,\ddot{\beta} - .7255\times10^{3}\,\beta\right] + 2\times79.86\,\theta
\end{aligned}
$$

$$
\begin{aligned}
\therefore\ &(1.0506\,D^{2} + .76283\,D + 21.903)\,\theta - (.0623\,D^{2} + .06696\,D - .4324\,K)\,\beta \\
&= M_0'\times10^{-6} - .0637\,\dot{\beta}^{2}\beta + .0134\,\dot{\theta}^{2}\theta \qquad\text{------------- (42)}
\end{aligned}
$$

Flap Equation of the Fighter

Eq. (14) after $\theta = \alpha$, becomes

$$
\begin{aligned}
&m(k_x^{2} + r^{2})\,\ddot{\beta} - m r (A + C)\,\ddot{\theta} + m(A + C)\,r\,\dot{\theta}^{2}\theta \\
&= L_2 + \Sigma L_w a_w + r\,\Sigma L_t
\end{aligned}
$$

$$
\begin{aligned}
&453.4\,[5.61^{2} + 18.3^{2}]\,\ddot{\beta} - 453.4\times18.3\,(3.25 + .5912)\,\ddot{\theta} + 453.4\,(3.25+.5912)\,18.3\,\dot{\theta}^{2}\theta \\
&= L_2 + 142.24\times10^{3}\times17.95\,\theta - 4.871\times10^{3}\times21.6\,\dot{\beta} + 59.73\times10^{3}\times18.3\,K\beta \\
&\quad + 18.3\,(8.784\times10^{3}\,\theta + 1.1325\times10^{3}\,\dot{\theta} + .03375\times10^{3}\,\ddot{\beta} - .7255\times10^{3}\,\beta)
\end{aligned}
$$

Simplifying.

$$(1.6571\,D^2 + 1.184\,D - 10.93K)\beta - (.31913\,D^2 + .2059\,D + 27.13)\,\theta$$
$$= L_2 \times 10^{-5} - 0.319\,\dot{\theta}^2\theta$$

$$- - - - - - - - (43)$$

The characteristic equation of the system is

$$\begin{vmatrix} 1.0506\,D^2 + .76283\,D + 2.1903 & .0623\,D^2 + .06696\,D - 4.324\,K \\ .31913\,D^2 + .2059\,D + 27.13 & 1.6571\,D^2 + 1.184\,D - 10.93\,K \end{vmatrix}$$

or

$$D^4 + 1.437\,D^3 + (1.643 - 6.592K)\,D^2 + (.451 - 4.793K)\,D - 7.095K = 0$$

The Routh Discriminant, 1.437 (1.643 -6.592K) -.451 + 4.793K > 0 gives

$$K < .408$$

Since the last coefficient in the polynominal requires that K be negative, the system is stable for all negative values of K.

2. Yaw Stability

Degree of Freedom:

Yaw Angle = ψ

The equation of motion for the case with the freedom of yaw restrained may be obtained from Eq. (16) by setting $\gamma = 0$.

$$\left[I_{zz} + 2m(k_z^2 + \overline{R+r}^2) \right] \ddot{\psi} = N_z + \Sigma N_{\dot{\psi}} \dot{\psi} + \Sigma N_{\psi} \psi$$

Referring to Eq. (38)

$$(10.12D^2 + .532D + 4.725)\psi = N_o \times 10^{-6} \qquad ----- (44)$$

The system is evidently stable. The damping factor and the circular frequently may easily be found.

$$\text{Damping Factor, } a = -\frac{.532}{10.12} = -.05256$$

$$\text{Circular Frequency, } \omega = \sqrt{\frac{4.725}{10.12} - \left(\frac{.532}{10.12}\right)^2} = .68$$

$$\text{Period, } P = 2\pi/\omega = 9.2 \text{ sec.}$$

Conclusions:

1. Stable flight is possible with wing-tip attached F-84 fighters on B-50 bomber provided proper auto-pilot controls are incorporated in design.

2. In the single-joint attachment, the auto-pilot or manual control in the bomber was found to be required to acquire stability of flight. It is possible that only elevator movement be used for control in the fighters. However, this elevator deflection should respond to both the amplitude and the rate of change of flapping angle of the fighter about the attachment joint.

3. In the two-joint attachment, the stability is inherently more stable. Controls in the fighter may be provided by either movement of the redesigned flaps or flaps plus spoilers or symmetrical movement of the ailerons of the fighters.

4. Rudder control in the fighters is not required in either method of attachment.

5. The effect of c.g. locations of the fighters and of the bomber relative to axes of rotation of the fighter is not small. For the practical range of c.g. location, stable flight can be expected. It is advisable that these relative distances be made as small as practicable so as to reduce the effect of dynamic coupling.

6. It is pointed out that flexibility of the wing structure may prove important. Therefore, its effect should be considered.

APPENDIX A

BOEING AIRPLANE COMPANY
Seattle Division
Seattle 14, Washington

In Reply Refer To
Reference No.

E751-1502

Republic Aviation Corporation
Farmingdale, New York

Attention: Mr. R. G. Melrose

Subject: Fighter Tow Data

Gentlemen:

On February 3, 1949, at a conference on the subject program, Mr. D. W. Finlay, representing the Boeing Airplane Company, offered certain data and drawings on the B-50 airplane to assist Republic Aviation Corporation in the analysis of the matching of the F-84 and B-50 in a fighter-bomber team. We regret the delay in furnishing these data to you, and hope they will prove adequate for the purpose intended.

The data you requested is furnished in the following paragraphs and in Enclosure A.

(a) Weight and C.G. positions:

	Wt.	c.g. in % Mac. (Wheels Up)
Weight less fuel and bombs includes all Wibac modifications	93,798	28.6
Add 10,000 lbs. bombs	103,798	19.2
Add all fuel except bomb bay fuel	160,528	23.6
Add all fuel including bomb bay fuel	173,668	29.9

Republic Aviation Corp.
Page Two

Reference
E751-1502

(b) Speed and altitude pattern for typical missions:

Weight	Cruising Altitude	True Airspeed for Best Range
170,000	10,000	235 Knots
135,000	"	214 "
100,000	"	186 "
160,000	20,000	260 "
135,000	"	246 "
100,000	"	220 "
140,000	30,000	285 "
100,000	"	260 "

(c) Moment of inertia in pitch and roll:

Design Gross = 120,000 lbs.

c.g. = Sta 435.3

1/4 Chord (Mac) = Sta 430.1

Moments of inertia with respect to mutually perpendicular axes at 1/4 chord (Mac)

Pitching

I_{y-y} = 1000 slug ft^2

Rolling

I_{x-x} = 1712 slug ft^2

Yawing

I_{z-z} = $I_{x-x} + I_{y-y}$ = 2712 slug ft^2

(d) Aerodynamic damping coefficient in roll = - .0019 δ_a , where δ_a is aileron angle in degrees. This coefficient is defined as the rate of change of rolling moment coefficient, c_ℓ with the helix angle, $\frac{pb}{2v}$.

(e) Aileron effectiveness parameter in the cruise condition, flaps up:

$$\frac{pb}{2v} = .0035\ \delta_a$$

(f) The maximum allowable aft c.g. position on B-50 airplanes is 34% mac. The c.g. position for neutral stability varies with flap position, power, and speed from about 34% to a much further aft c.g. position.

(g) Airplane damping moment coefficient in pitch = 93.5 $\frac{\theta}{V}$, where θ = angular pitch velocity in radians per second and V = airspeed in mph.

(h) Rather than attempt to show what loads can be allowed on the wing tip, the strength of the wing at a number of spanwise stations is shown on the page from Document D-7051 in Enclosure A. Also included are the distribution of wing dead weights and the shears, moments, and torsions due to dead weight at a unit load factor for weight condition D which is a minimum flight weight condition and for weight condition B in which condition the wing contains a maximum quantity of fuel. In addition to the weights included in condition B, it is possible to add an external tank at sta. 533. The tank and fuel weigh approximately 4600 pounds. Air load information is not included due to the unknown span loading in the coupled airplane configuration. Drawings furnished are adequate for determining locations of wing stations and loaded areas for calculation of air load shears, moments, and torsions.

(i) Drawings of the outer wing are included in Enclosure A.

(j) It is physically possible to install a B-50 outer wing panel on a B-29 airplane though some rework of the structural connection and aileron control systems would be necessary. The B-29 wing allowable loads are approximately 20 percent less than for the B-50.

Very truly yours,

BOEING AIRPLANE COMPANY
SEATTLE DIVISION

Lysle A. Wood
Chief Engineer

Enclosure A: Drawings and
Data under Shipment Notice No. 16706

REFERENCES

1. Dr. Vogt, R. - "Escort of the B-50 by Two Fighters" Wright Field Report No. XXII.

2. Lu, H. R. - "Aerodynamic Interference Effect Between B-50 and Wing-Tip Attached F-84 Airplane" Republic Aviation Report No. EDR-F905-101

3. Tucker, W.A. & Goodson, K.W. - "Tests of a 1/5-Scale Model of the Republic XP-84 Airplane in the Langley 7x10 Ft Tunnel" -- NACA MR No. L6F25, July 17, '46 (RMR-61).

4. Uspensky, J.V. - "Introduction to Theory of Equation" McGraw Hill Company

TABLE I

SUMMARY OF DYNAMIC STABILITY STUDY

(No Automatic Controls in Bomber)

A	k_e	k_a	c	Roll Stability	Pitch Stability	Combined Stability	Combined Stability Region	Remarks
3.25	.1	0	-.3	Yes	Yes	Yes	K_a -.5 to .5 K_e 0 to .03	The effect of increasing "c" on combined stability area is small.
	.1	0	0	Yes	Yes	Yes	K_a -.8 to .6 K_e 0 to .05	
	.1	0	.5	Yes	Yes	Yes	K_a -.5 to .6 K_e 0 to .04	
	.1	0	1.0	Yes	Yes	Yes	K_a -.6 to .6 K_e 0 to .05	
	.1	-2	-.3	Yes	Yes	No	---	Adding a rate aileron gyro ($k_{\dot{a}}$) requires a displacement gyro (K_a) at c = .5, No comb. sta. for other c values.
	.1	-2	0	Yes	Yes	Yes	Very small	
	.1	-2	.5	Yes	Yes	Yes	K_a - 2 to -.5 K_e .03 to .07	
	.1	-2	1.0	Yes	Yes	No		
	.15	0	0	Yes	No	No		Increasing k_e from .1 to .15 gives no Comb. Sta.
	.15	-2	1	Yes	Yes	No		

TABLE I (cont'd)

A	k_e	k_a	c	Roll Stability	Pitch Stability	Combined Stability	Combined Stability Region	Remarks
325	.05	0	0	Yes	Yes	Yes	K_a -.7 to .3 K_e 0 to .04	Effect of decreasing k_e from .1 to .05 on combined stability is small.
	.05	0	1	Yes	Yes	Yes	K_a -1.0 to .2 K_e 0 to .07	
	.05	-2	0	Yes	Yes	No	---	
	.05	-2	1	Yes	Yes	Yes	K_a -2 to -.5 K_e .02 to .07	
0	.1	0	0	Yes	Yes	Yes	K_a -.5 to .6 K_e 0 to .03	Reducing A to 0 decreases Comb. Sta. Area.

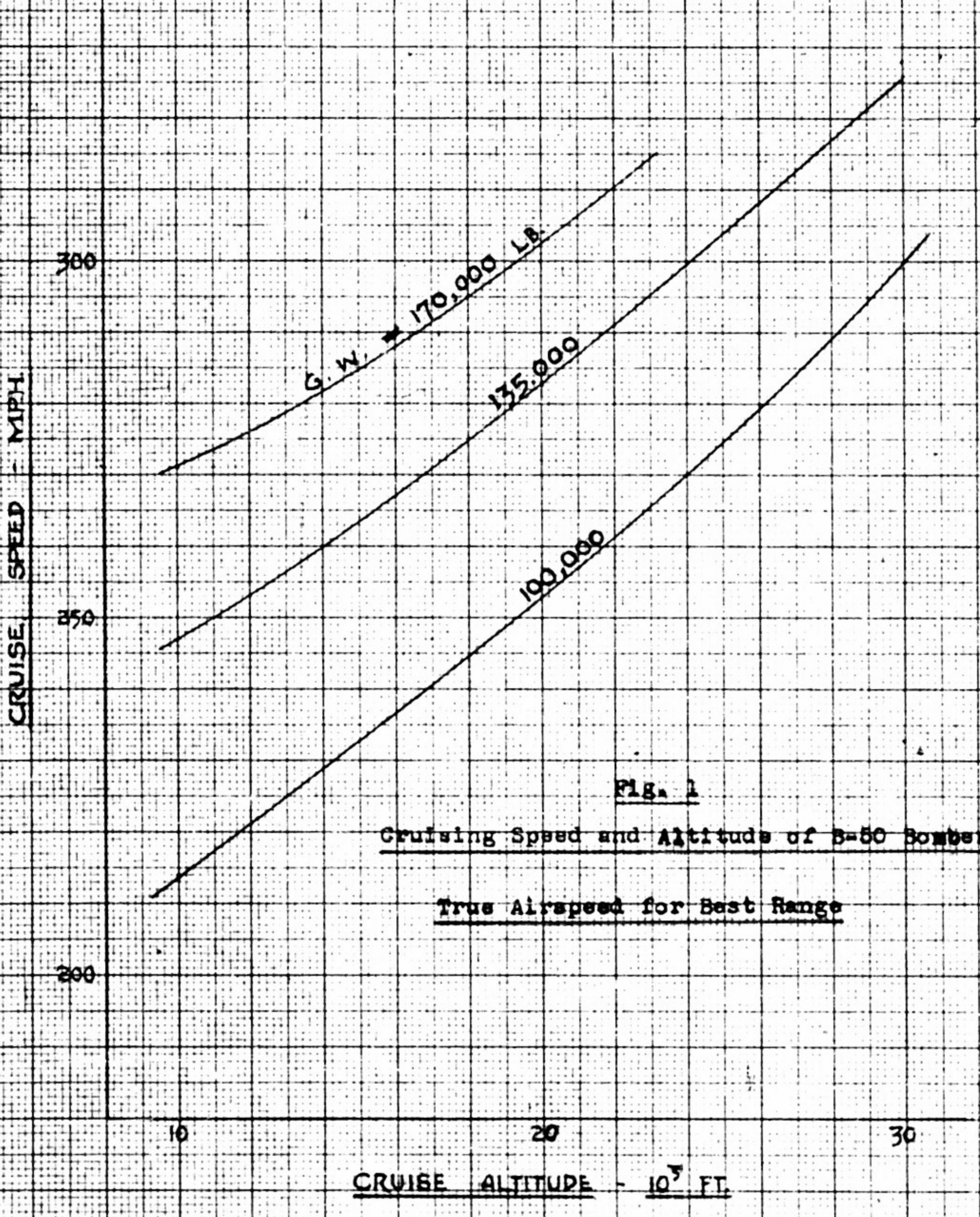


Fig. 1

Cruising Speed and Altitude of B-50 Bomber

True Airspeed for Best Range

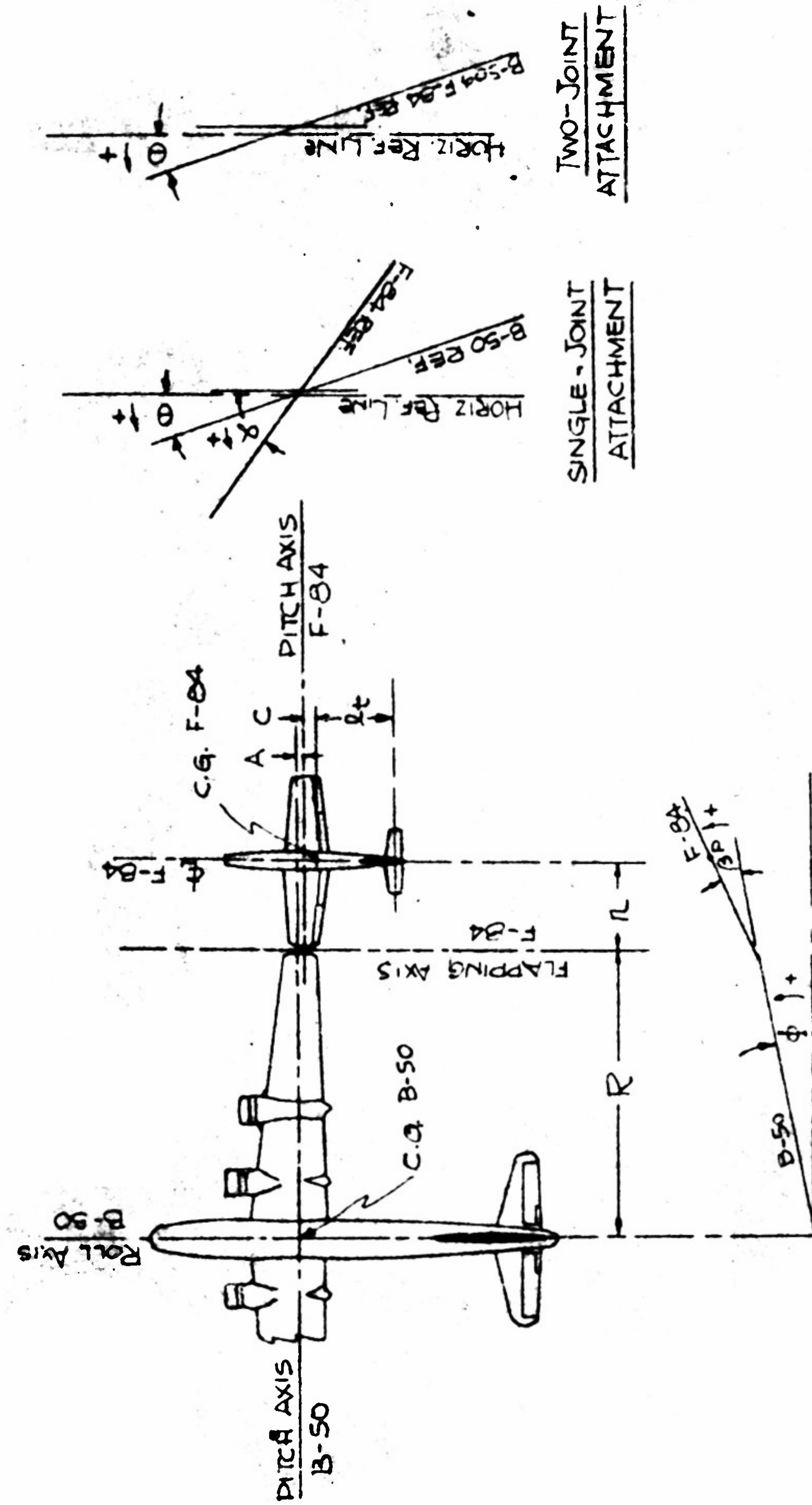


FIG. 2

PLANFORM – PROPOSED COUPLING SCHEMES

CONFI[DENTIAL]

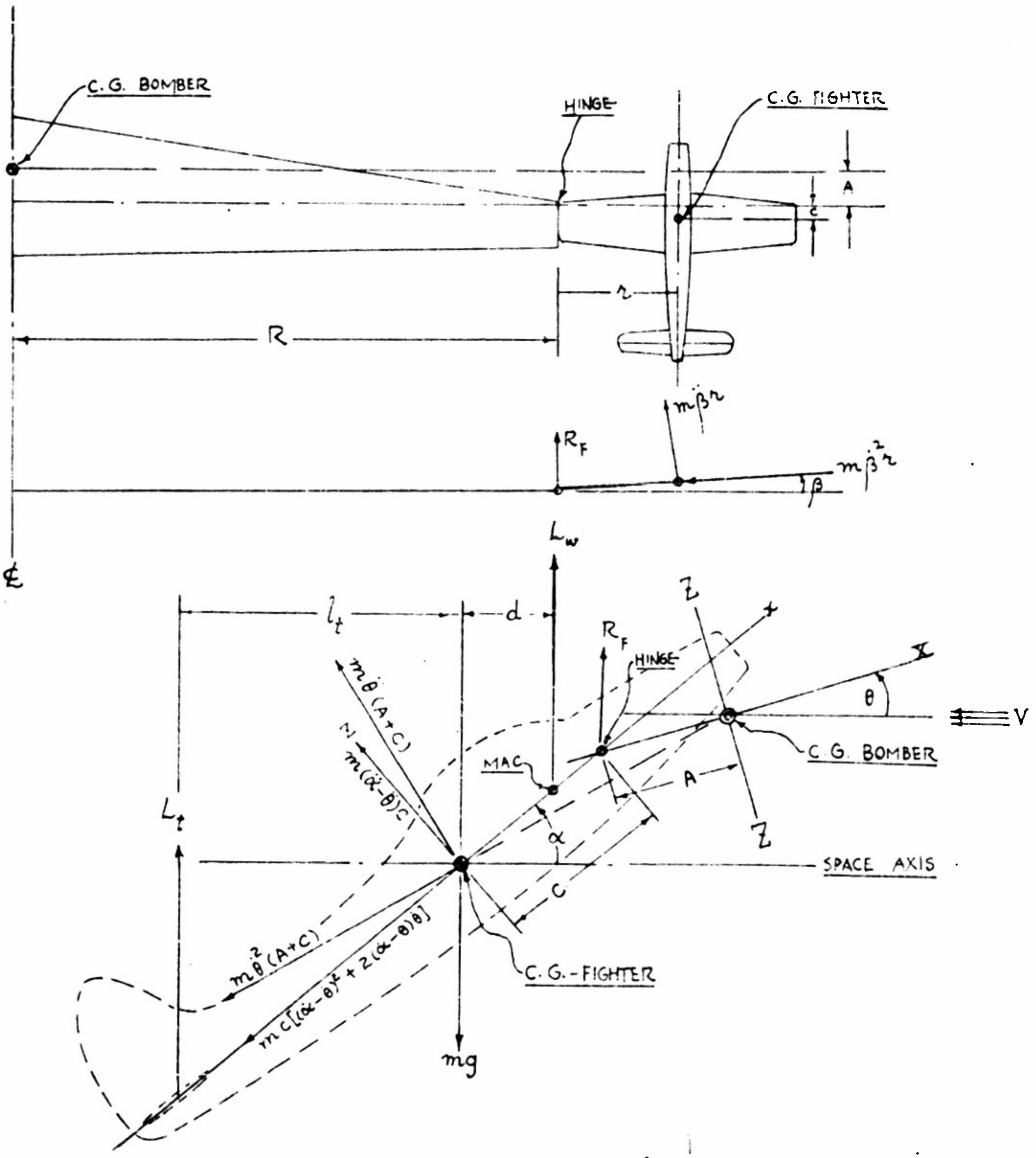


Fig. 3 - Force and Accerleration Diagram

Pitch Disturbance

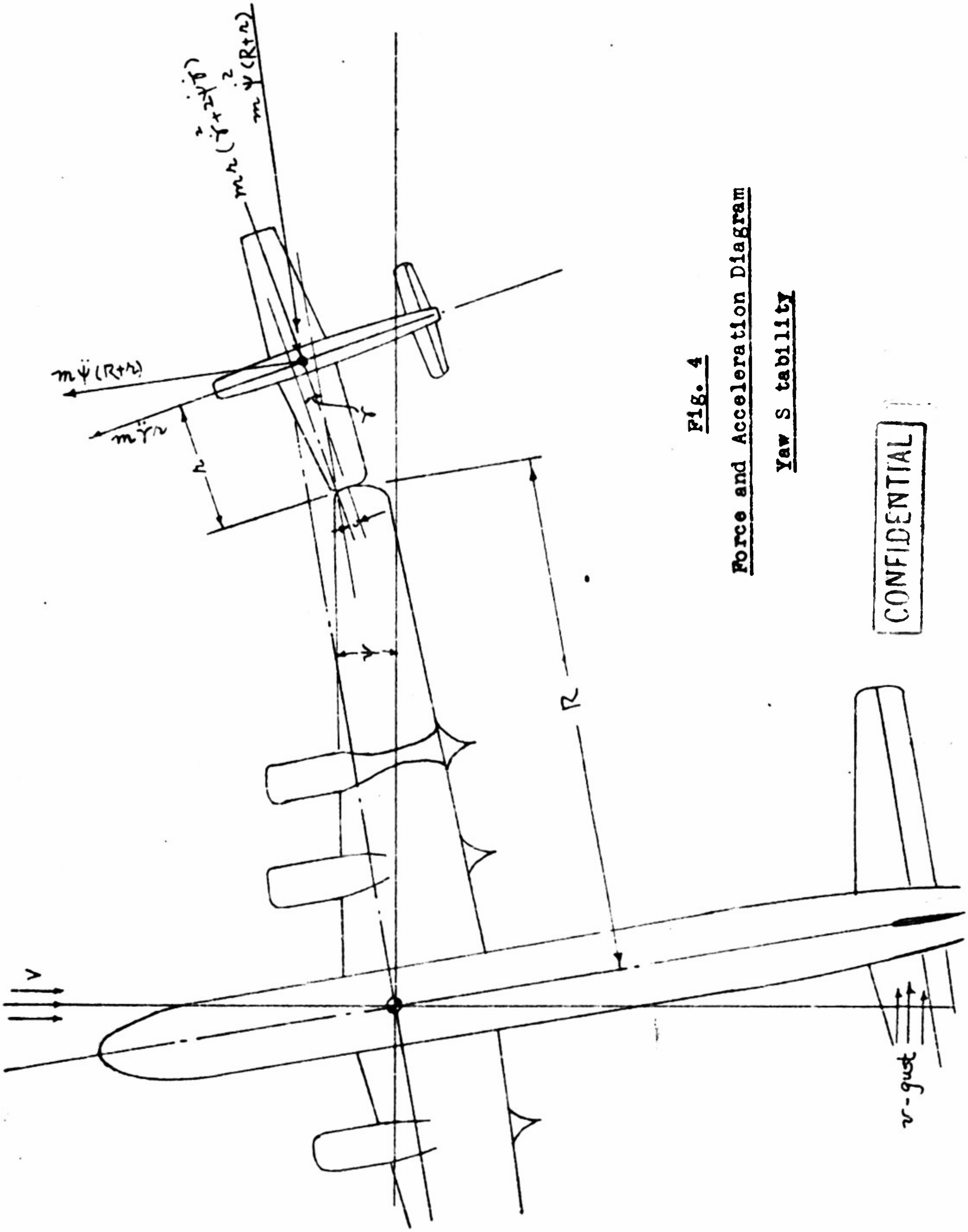


Fig. 4

Force and Acceleration Diagram

Yaw Stability

CONFIDENTIAL

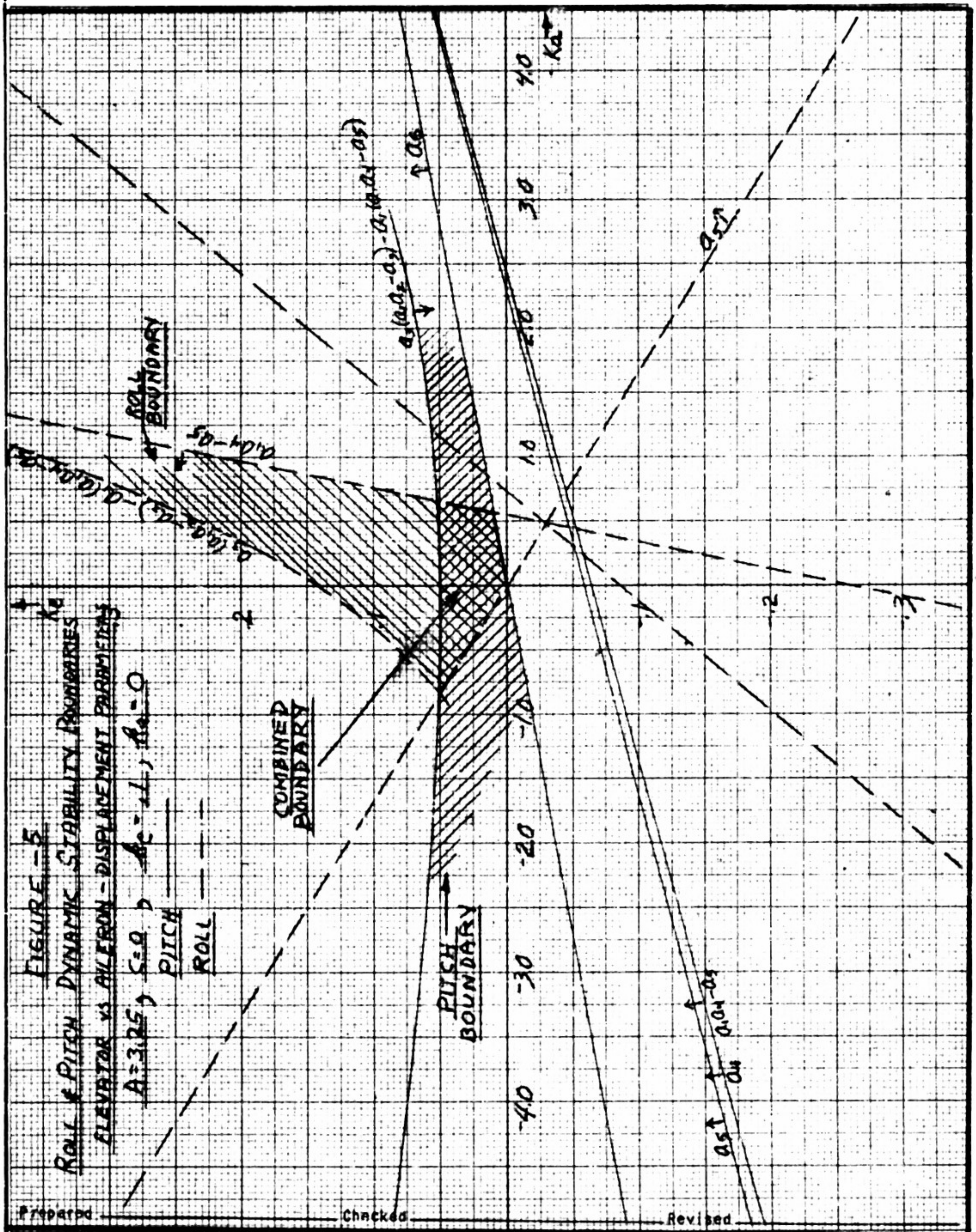
REPUBLIC AVIATION CORPORATION

Report No. EDR-F905-102

Page No. 70

Model

Date



Prepared

Checked

Revised

E-S-I-10A
REV. 4/49

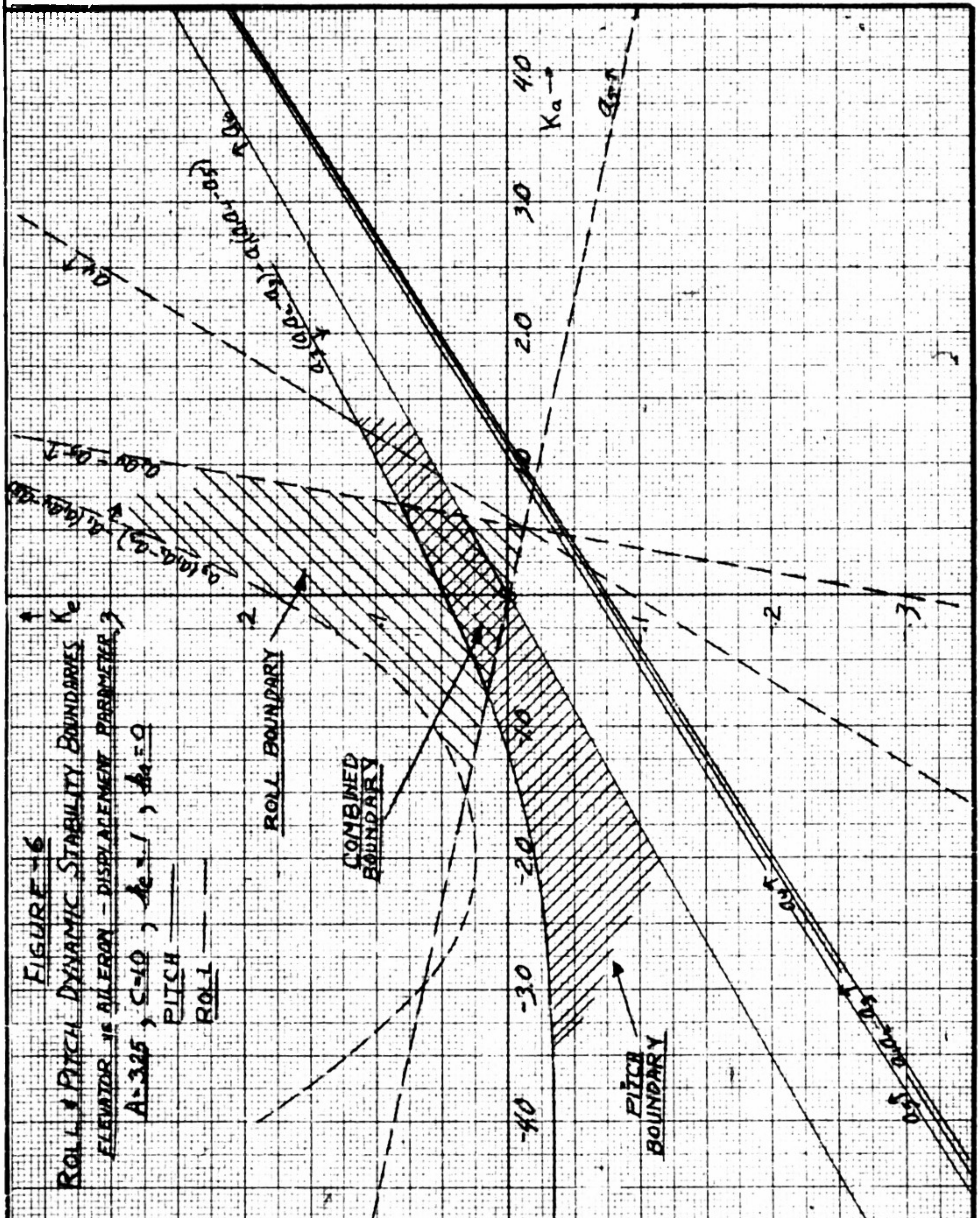
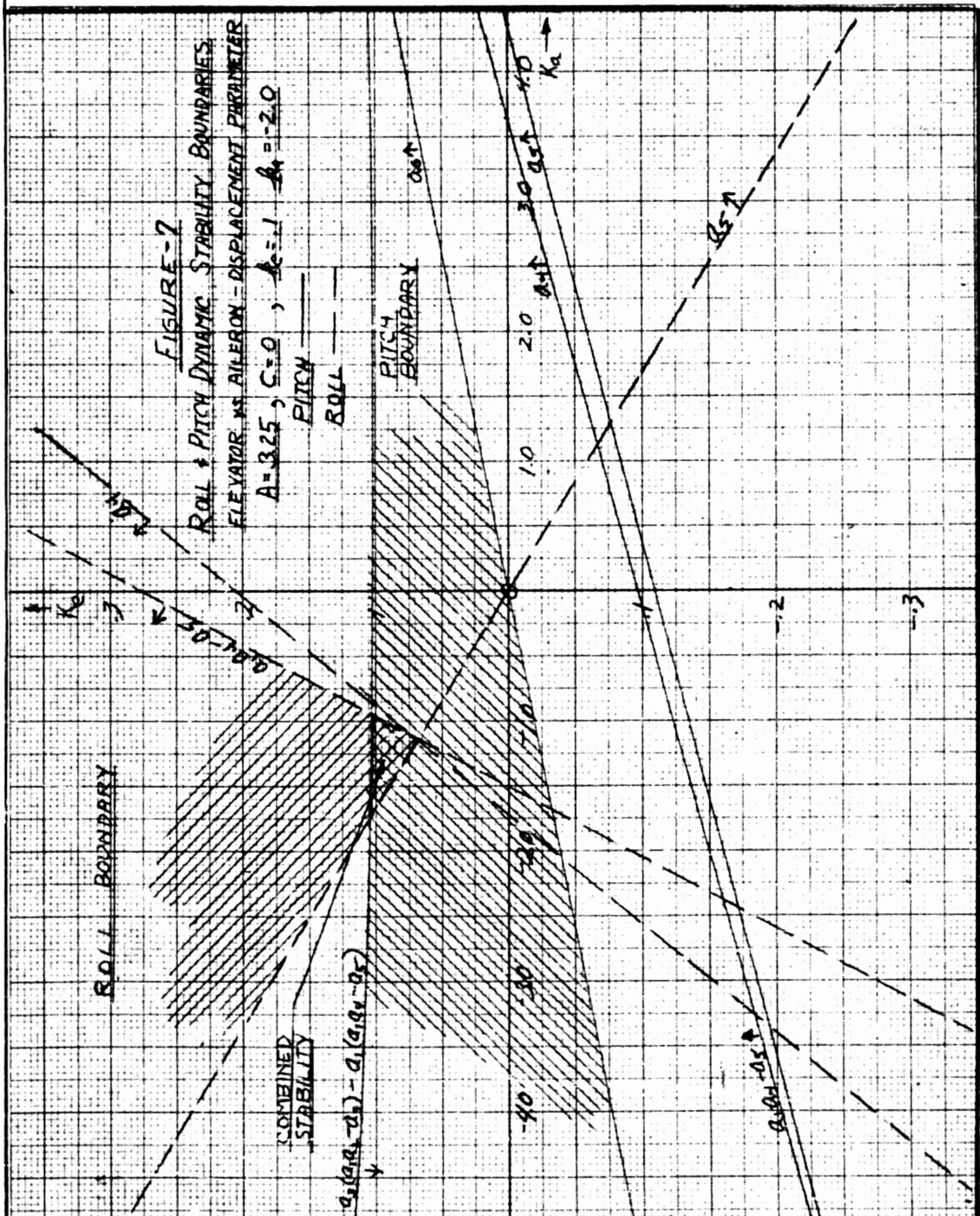


FIGURE - 6
ROLL & PITCH DYNAMIC STABILITY BOUNDARIES
ELEVATOR vs AILERON - DISPLACEMENT PARAMETER
A=3.25, C=10, ℓe=1, ℓa=0
PITCH
ROLL
ROLL BOUNDARY
COMBINED BOUNDARY
PITCH BOUNDARY
Ke
Ka
4.0
3.0
2.0
-1.0
-2.0
-3.0
-4.0
2
2
3

REPUBLIC AVIATION CORPORATION

Model

Report No. EDR-F905-102
Page No. 72
Date



Prepared
Checked
Revised

E-S-1-10A
REV.4/49

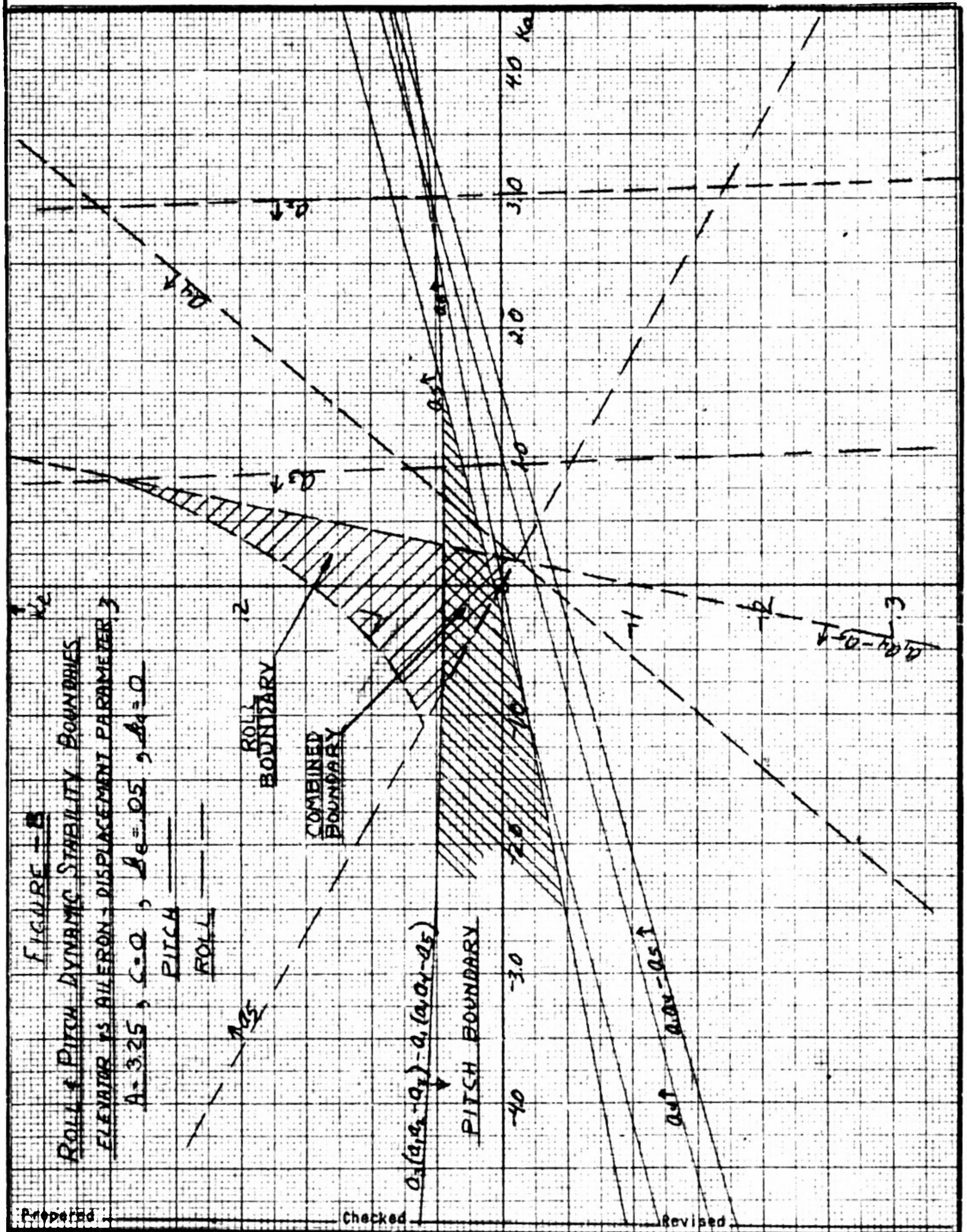


FIGURE - 8
ROLL & PITCH DYNAMIC STABILITY BOUNDARIES
ELEVATOR vs AILERON - DISPLACEMENT PARAMETER
A = 3.25, C = 0, β_e = .05, β_a = 0
PITCH
ROLL
ROLL BOUNDARY
COMBINED BOUNDARY
PITCH BOUNDARY

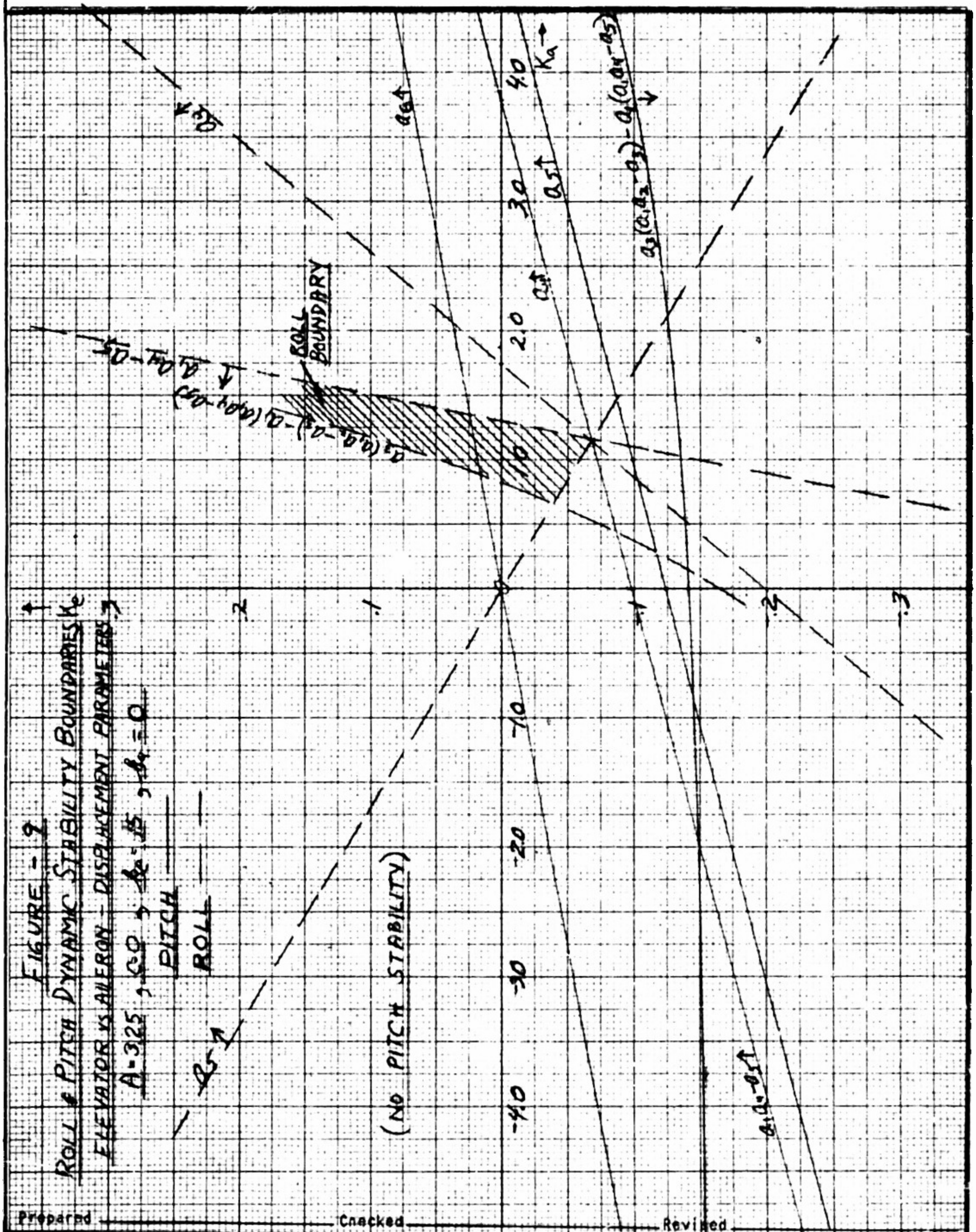


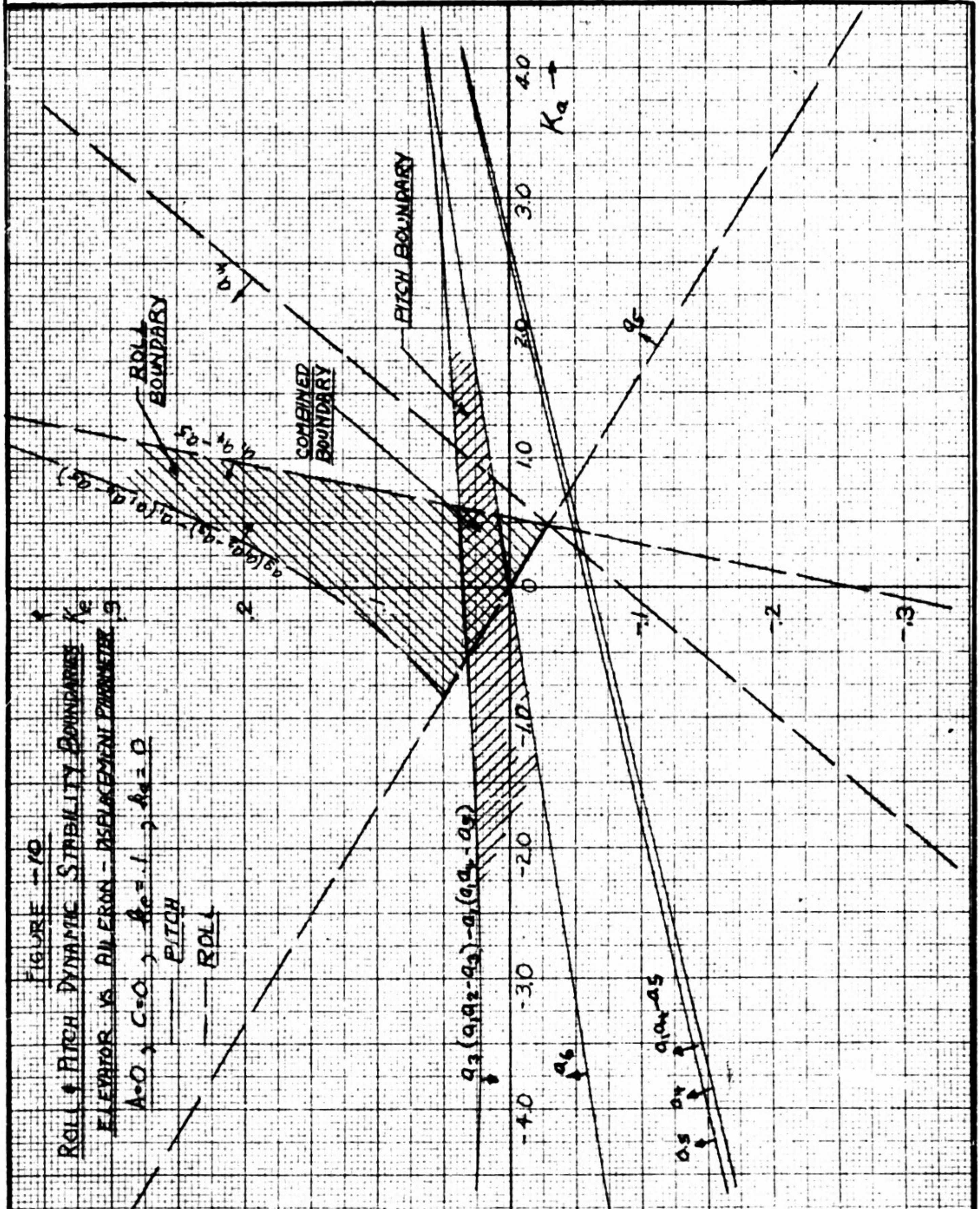
FIGURE - 9
ROLL & PITCH DYNAMIC STABILITY BOUNDARIES
ELEVATOR vs AILERON - DISPLACEMENT PARAMETERS
A=3.25, C=0, ke=.15, ka=0
PITCH
ROLL
(NO PITCH STABILITY)
ROLL BOUNDARY
Ke
Ka
4.0
3.0
2.0
1.0
-1.0
-2.0
-3.0
-4.0
1
2
3
-1
-2
-3

REPUBLIC AVIATION CORPORATION

CONFIDENTIAL

Model ____________

Report No. EDR-F905-102
Page No. 75
Date ____________



Prepared ____________ Checked ____________ Revised ____________

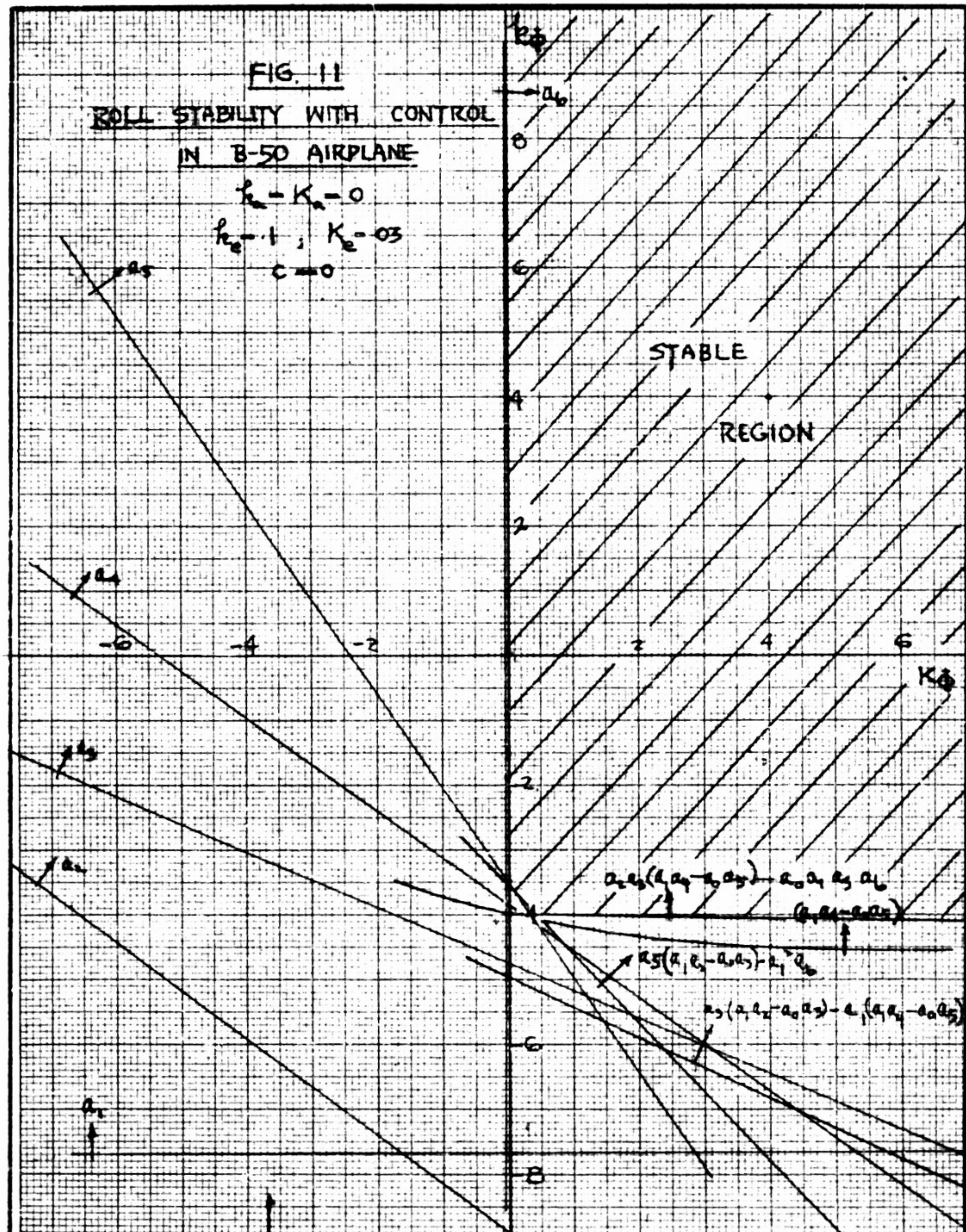
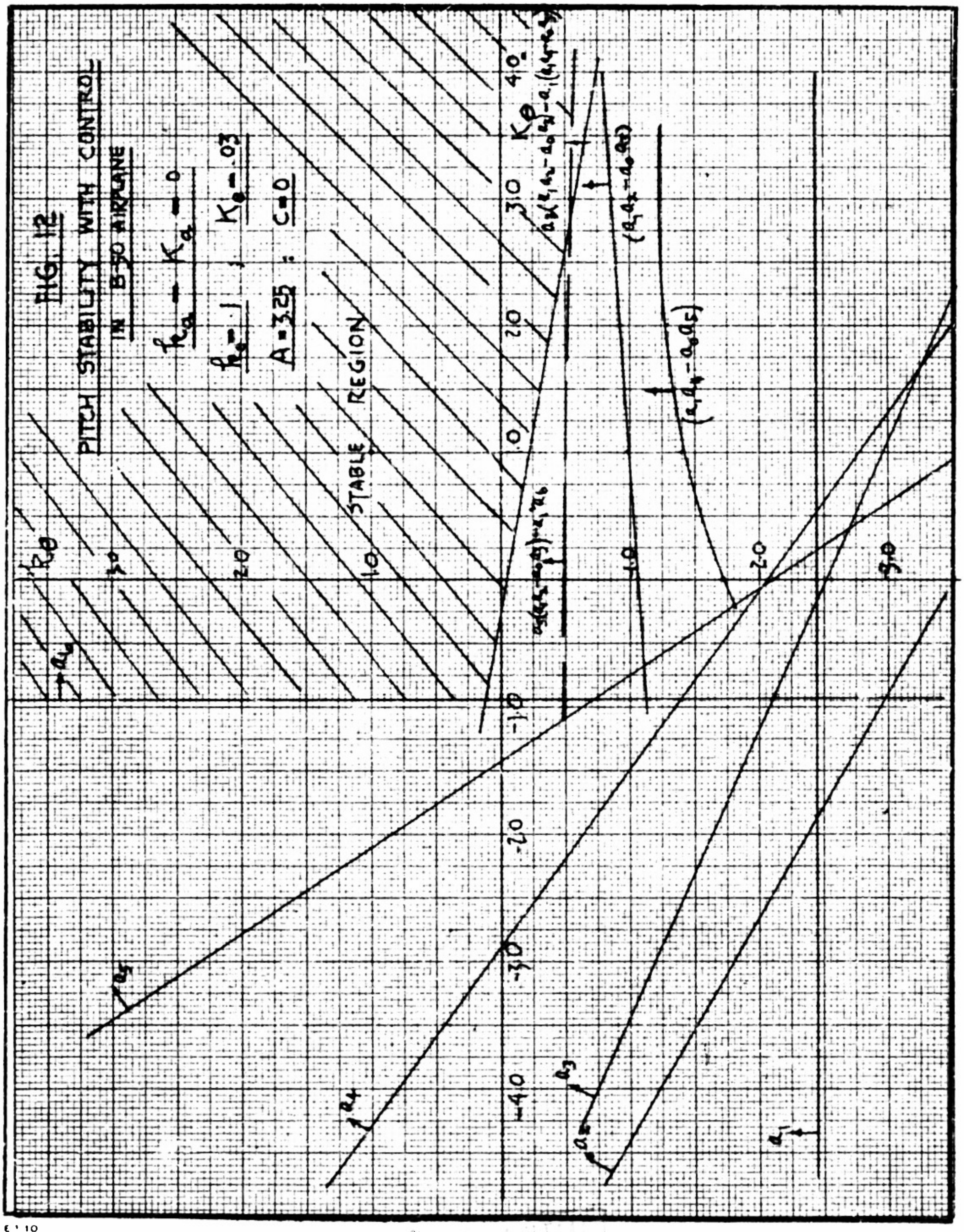


FIG. 11
ROLL STABILITY WITH CONTROL IN B-50 AIRPLANE
$k_a = K_a = 0$
$k_e = .1$; $K_e = .03$
$c = 0$
k_Φ
a_6
STABLE
REGION
K_Φ
a_5
a_4
a_3
a_2
a_1
$a_2 a_3(a_1 a_4 - a_0 a_5) - a_0 a_1 a_5 a_6$
$(a_1 a_4 - a_0 a_5)$
$a_5(a_1 a_2 - a_0 a_3) - a_1^2 a_6$
$a_3(a_1 a_2 - a_0 a_3) - a_1(a_1 a_4 - a_0 a_5)$
$(a_1 a_2 - a_0 a_3)$

PREPARED ____________ **REPUBLIC AVIATION CORPORATION** PAGE EDR-F905-102

CHECKED ____________ FARMINGDALE, L. I., NEW YORK REPORT NO. 77

REVISED ____________ MODEL ____________



E 10

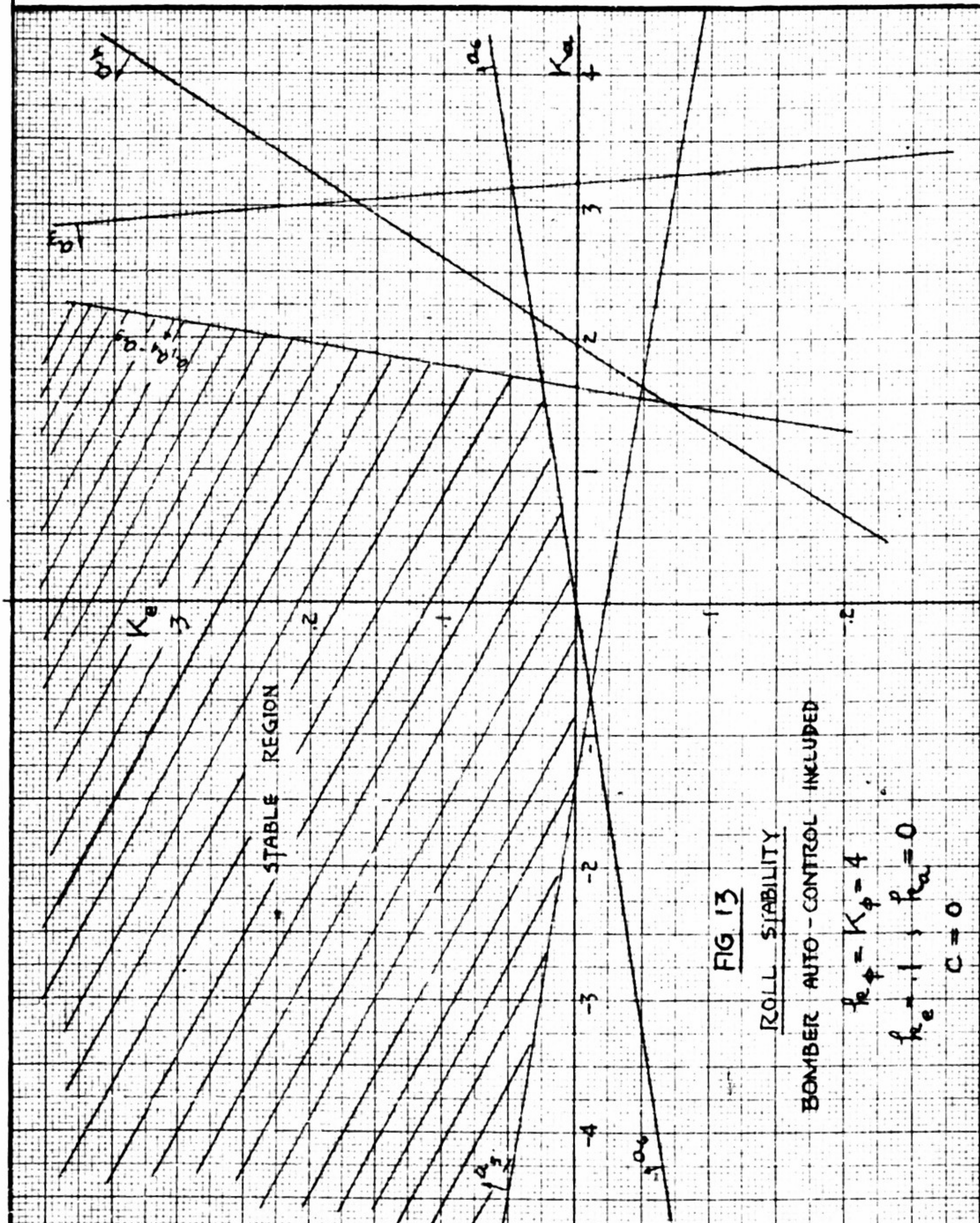


FIG 13

ROLL STABILITY

BOMBER AUTO-CONTROL INCLUDED

$k_\phi = K_\phi = 4$

$k_e = 1 \;;\; k_a = 0$

C = 0

Prepared ________ Checked ________ Revised ________

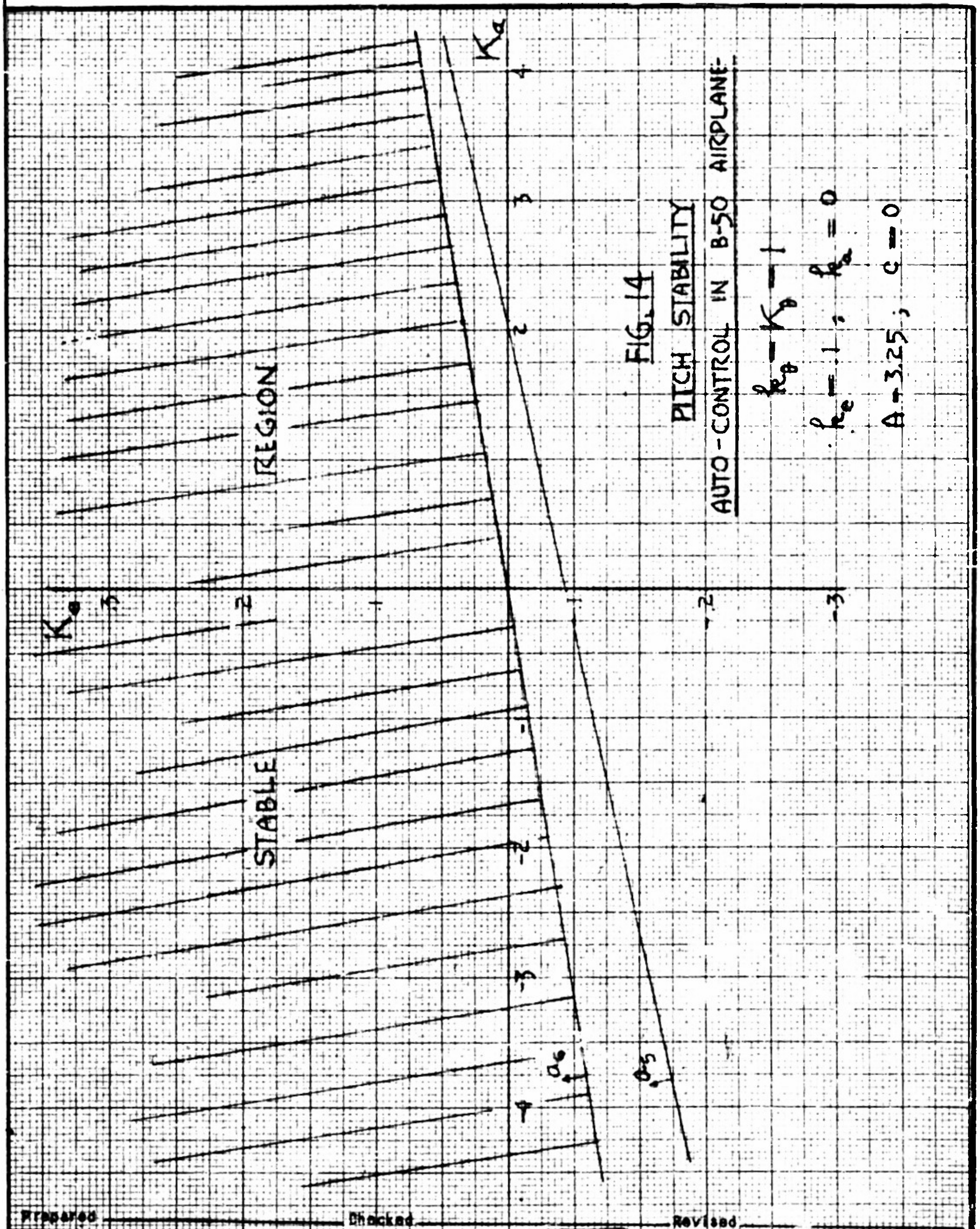


FIG. 14
PITCH STABILITY
AUTO-CONTROL IN B-50 AIRPLANE
k_θ = K_θ = 1
k_e = 1 ; k_α = 0
A = 3.25 ; C = 0
STABLE REGION
K_α
K_e
a_6
a_5
1
2
3
4
-1
-2
-3
-4
.1
.2
-.1
-.2
-.3

AD-A800 180

(UNPUBLISHED CARD) See Reverse ~~CONFIDENTIAL~~ (CORR. COPY 1) U

ATI 93 784 (COPIES OBTAINABLE FROM CADO) 302 300

REPUBLIC AVIATION CORP., FARMINGDALE, L.I., N.Y.
(REPORT NO. EDR-F905-102)

THEORETICAL INVESTIGATION OF THE DYNAMIC STABILITY OF BOMBER-FIGHTER COUPLED FLIGHT - AND APPENDIX A

H.R. LU; S. HELFMAN; J. CANGELOSI 10 JULY'49
79PP DIAGRS, GRAPHS

USAF PROJECT MX-1016

AERODYNAMICS (2)
STABILITY AND
CONTROL (1)

AIRPLANES - DYNAMIC STABILITY
AIRPLANES - WING-TIP COUPLING
PROJECT MX-1016

CO~~NFIDENTIAL~~